The compensation for risk in credit markets

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Overview of the talk

- Risk premia in corporate bond markets
- Risk premia in CDS and stock option markets

What's in a credit spread?

- Consider a world without taxes and with perfectly liquid markets
- Suppose that default risk is completely diversifiable: objective (P)= risk neutral (Q) survival rates
- Assume P= 90%, zero recovery and r=5%. What is the bond yield (and spread)?

$$B = \frac{0.9 \cdot 100}{1.05} = 85.71 \qquad \frac{100}{1+y} = 85.71 \rightarrow y = 0.1667$$

Systematic default risk

- So a positive spread over the risk free rate does not mean there is a premium for default risk - just compensation for expected losses.
- Suppose now that default risk is systematic and as a result there is a default risk premium
- This will translate into a lower risk-adjusted survival probability than the objective ($Q<P$) 0.9, say 0.8. So the bond price would be

$$B = \frac{0.8 \cdot 100}{1.05} = 76.16 \qquad \frac{100}{1+y} = 76.16 \rightarrow y = 0.3130$$

Expected loss / Risk premia (EL / RP)

- So the total spread of 26.3% consists of

  11.67% compensation for expected losses (EL spread)
  14.63% default risk premium (RP spread).

- Measuring these two components separately with and without corporate bond prices is part of what we do now ("Time varying risk premia in corporate bond markets" which is joint with Redouane Elkamhi).

Why is this important?

- Asset allocation (across products / over the cycle).
    - bonds with the same rating / default rate can have very different spreads depending on the systematic nature of their default risk.
    - Bonds / CDS across rating categories appear to have different mixes of expected losses / risk premia.
    - the same is true for multi-name tranched products. Equity tranches may have more risk in an absolute sense but super senior tranches should compensate more for systematic risk than expected losses.

How we compute risk premia

$$
\begin{aligned}
B^P_{t,T} &= \sum_{i=1}^{N} d_i \cdot c_i \cdot (1 - P_t(\tau < s_i)) + d_N \cdot p \cdot (1 - P_t(\tau < T)) \\
&\quad + R \cdot p \cdot \int_t^T d_s \cdot dP_t(s) \qquad \longrightarrow y^P
\end{aligned}
$$

$$
\begin{aligned}
B^Q_{t,T} &= \sum_{i=1}^{N} d_i \cdot c_i \cdot (1 - Q_t(\tau < s_i)) + d_N \cdot p \cdot (1 - Q_t(\tau < T)) \\
&\quad + R \cdot p \cdot \int_t^T d_s \cdot dQ_t(s) \qquad \longrightarrow \left(y^{Q,model}, y^{Q,market}\right)
\end{aligned}
$$

How do we compute default probabilities (P)?

- As an illustration consider Merton (1974)
- The Q-probability of default is $$1 - N\left(d_2\right)$$

$$d_2 = \frac{ln\frac{v_t}{F} + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

- and the P-probability would be $$1 - N\left(d_2^P\right)$$

$$d_2^P = \frac{ln\frac{v_t}{F} + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

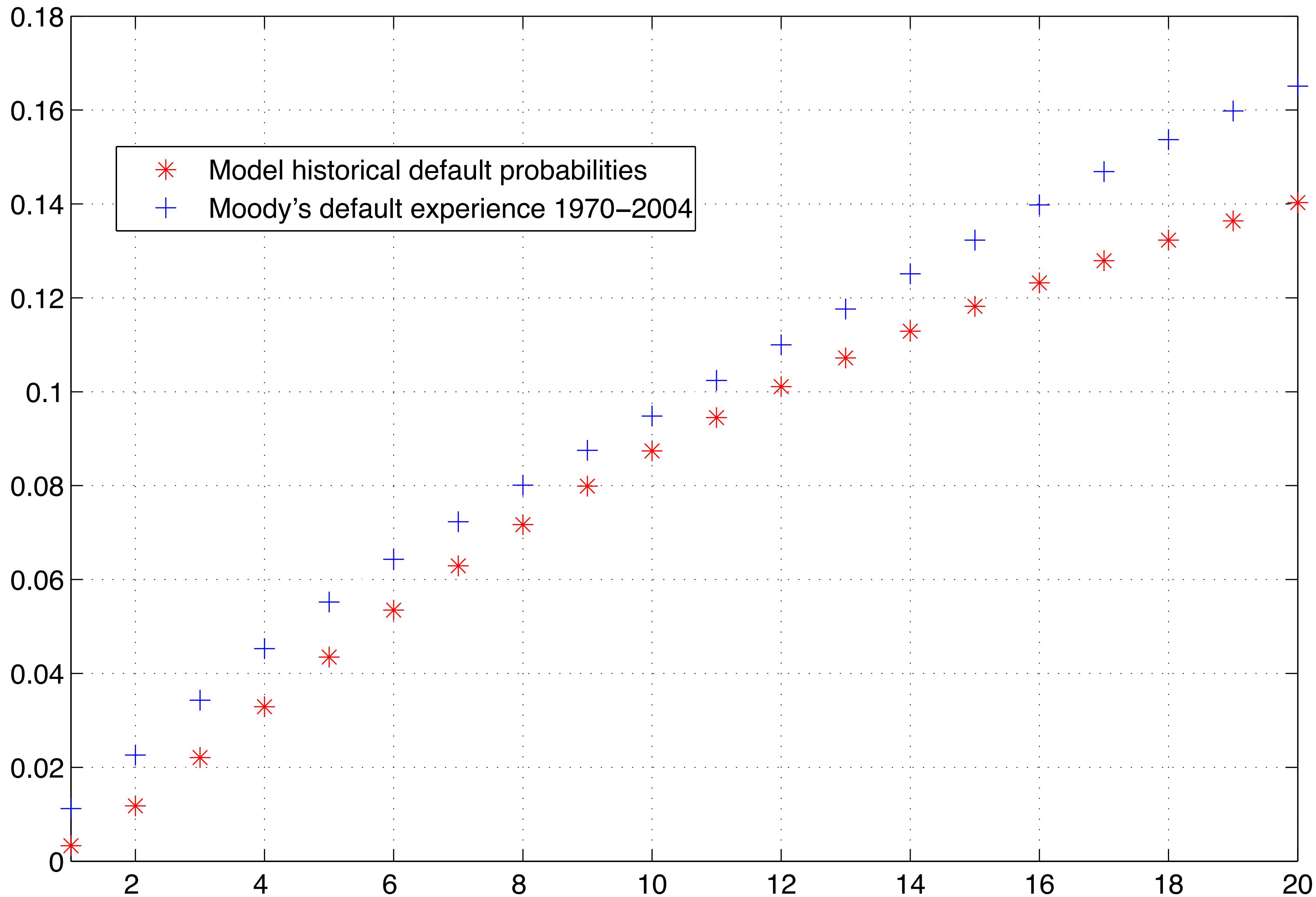
Estimating the asset drift

we use historical returns for this - ideally one would use a forward looking metric

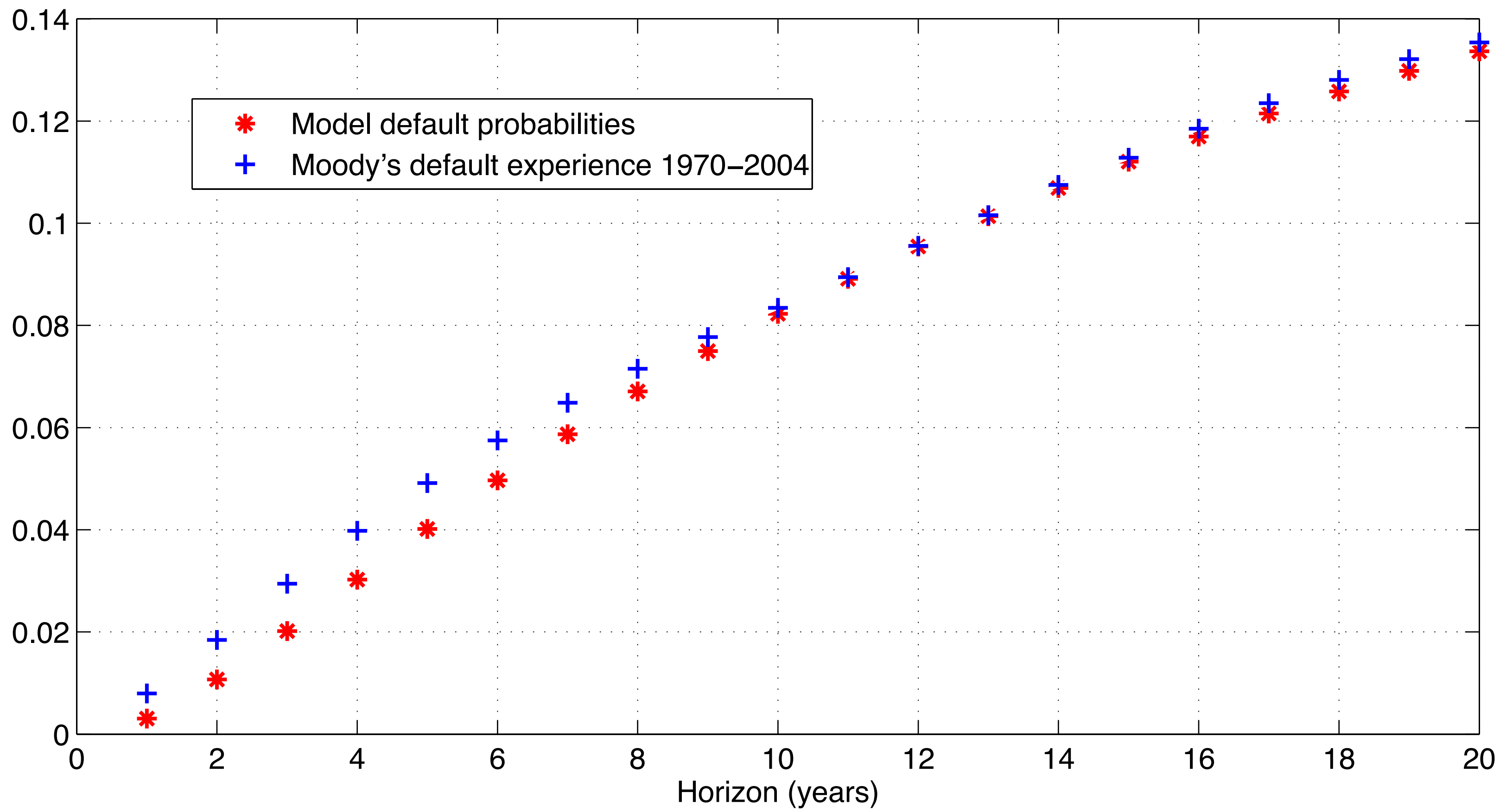
$$\mu_v - r = (R_v(t) - r) = \Delta_E \cdot (R_E(t) - r)$$

This is where the model comes in, we chose Leland & Toft (1996)

Our default probabilities



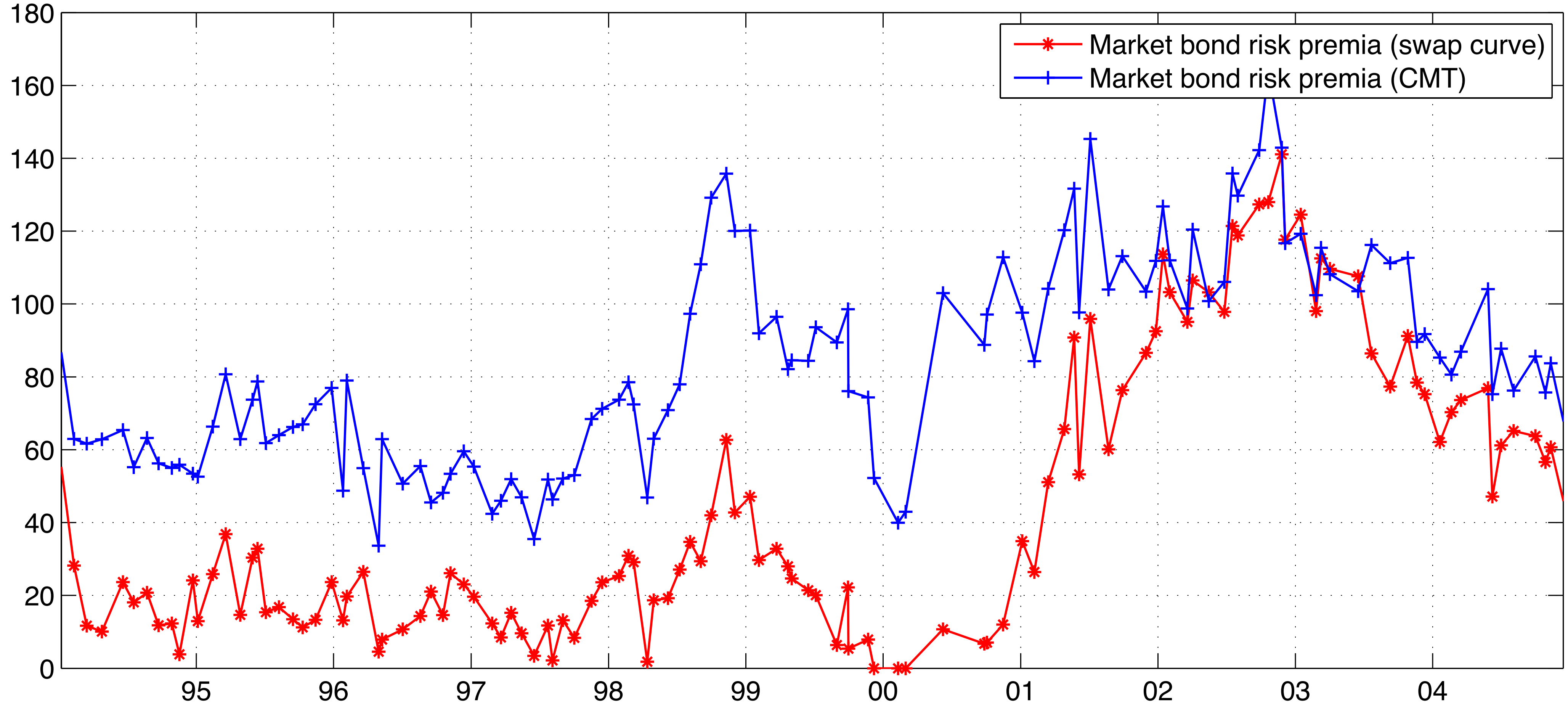
97% of the data (excluding AAA, CCC and less)

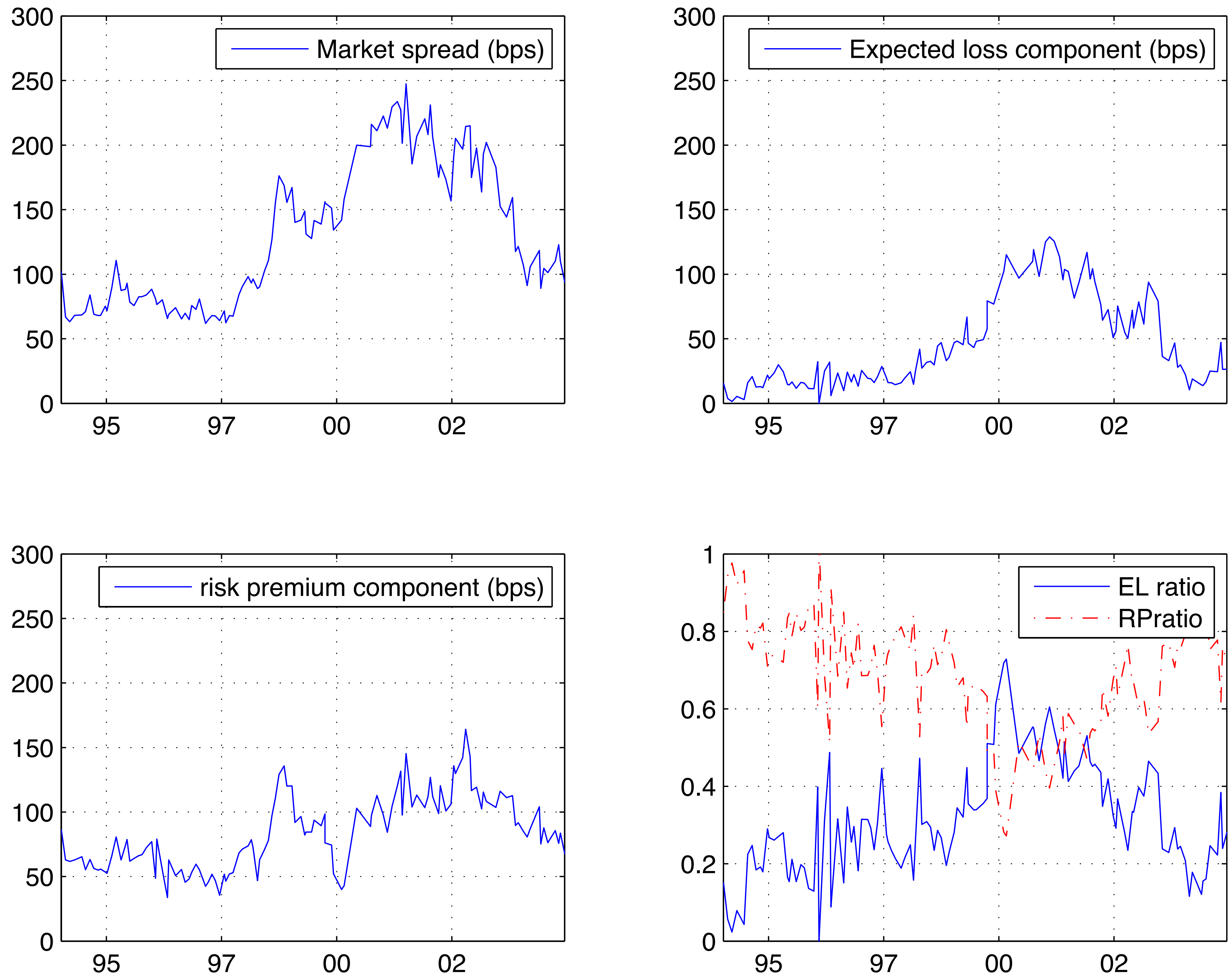


Our findings - preview

- risk premia are highly time-varying
    - they are similar to what has been found in CDS markets during the 2001-2004 period.
- Expected losses and risk premium spread components behave differently.
- Risk premia in credit and equity markets look very different -
    - a link can only be established by accounting for the non-linearity and dependence on time varying parameters.

Risk premia measured in bond markets

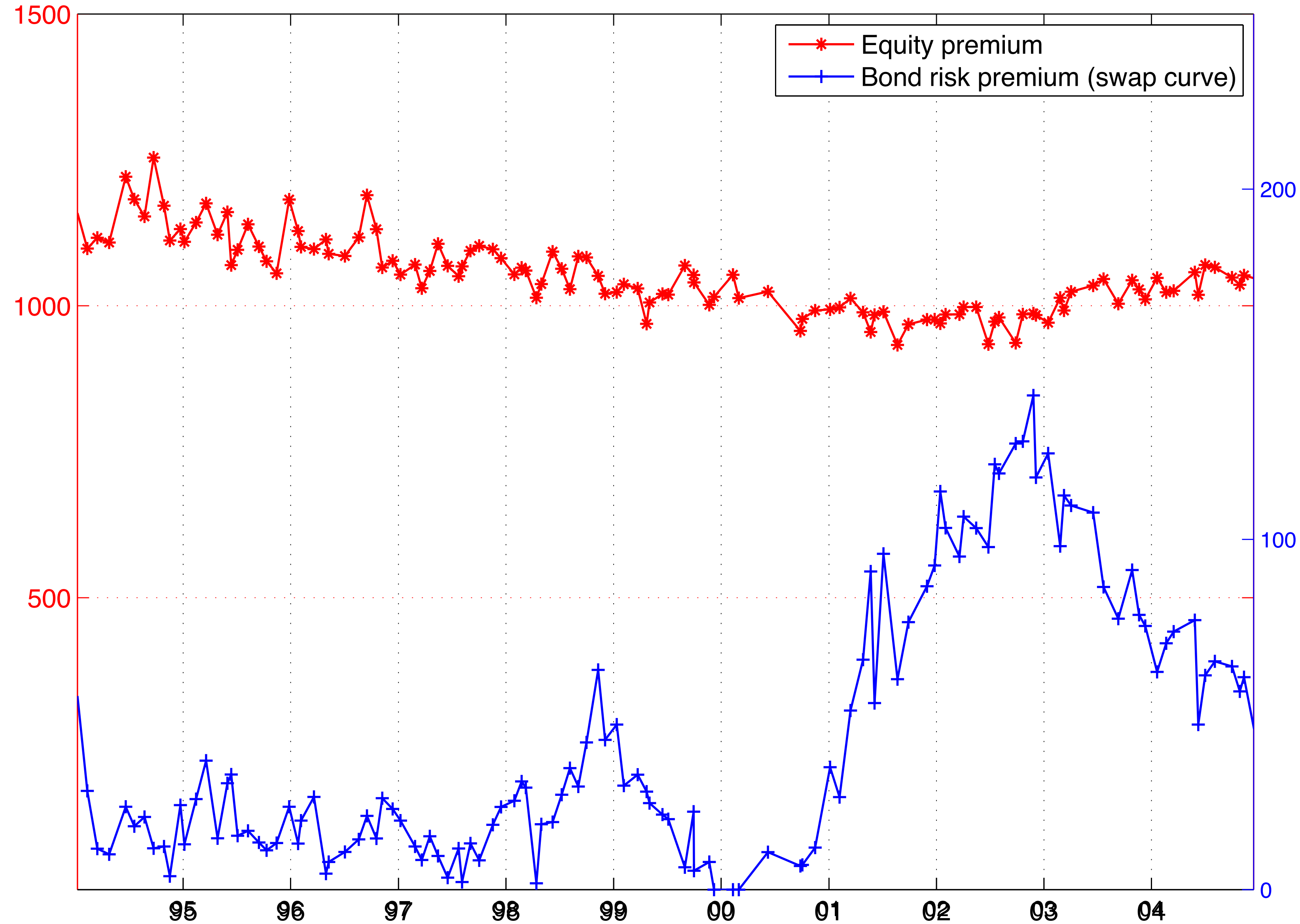



Market spread (bps)
Expected loss component (bps)
risk premium component (bps)
EL ratio
RPratio
95
97
00
02

Expected losses

- Elton et al. (2001, JF) find using historical rating based default rates that expected losses constitute about 25% of spreads on average.
- Our average is very close to this - but there is significant time variation: near zero lows and highs in the 70-80% range.
  - using the average may not be sufficient in applications

The link between risk premia in equity and credit markets is non-linear



and depends on time varying parameters.

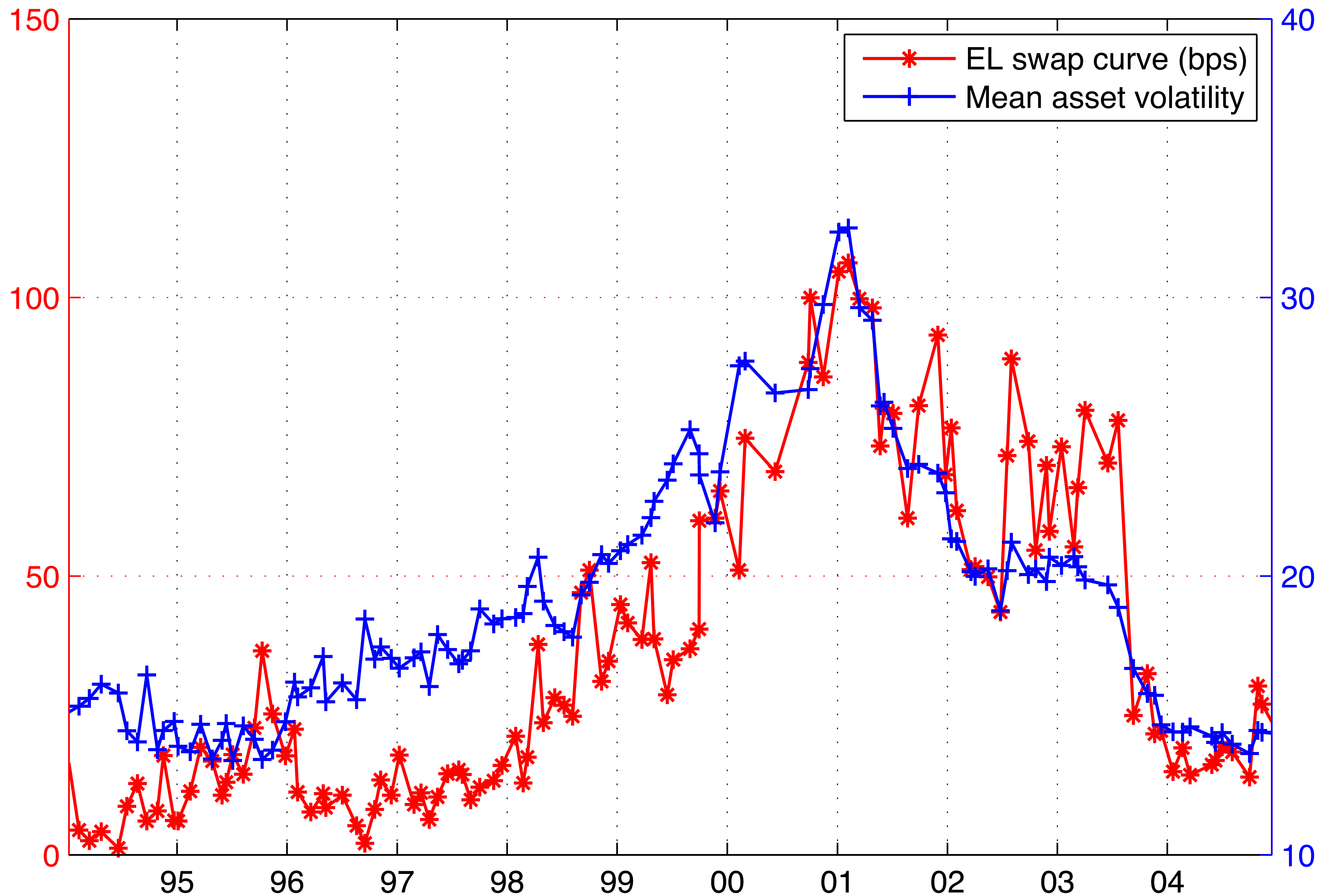
Basic corporate finance tells us (MM prop II) that

$$R_D = R_v + (R_v - R_E)\frac{E}{D}$$

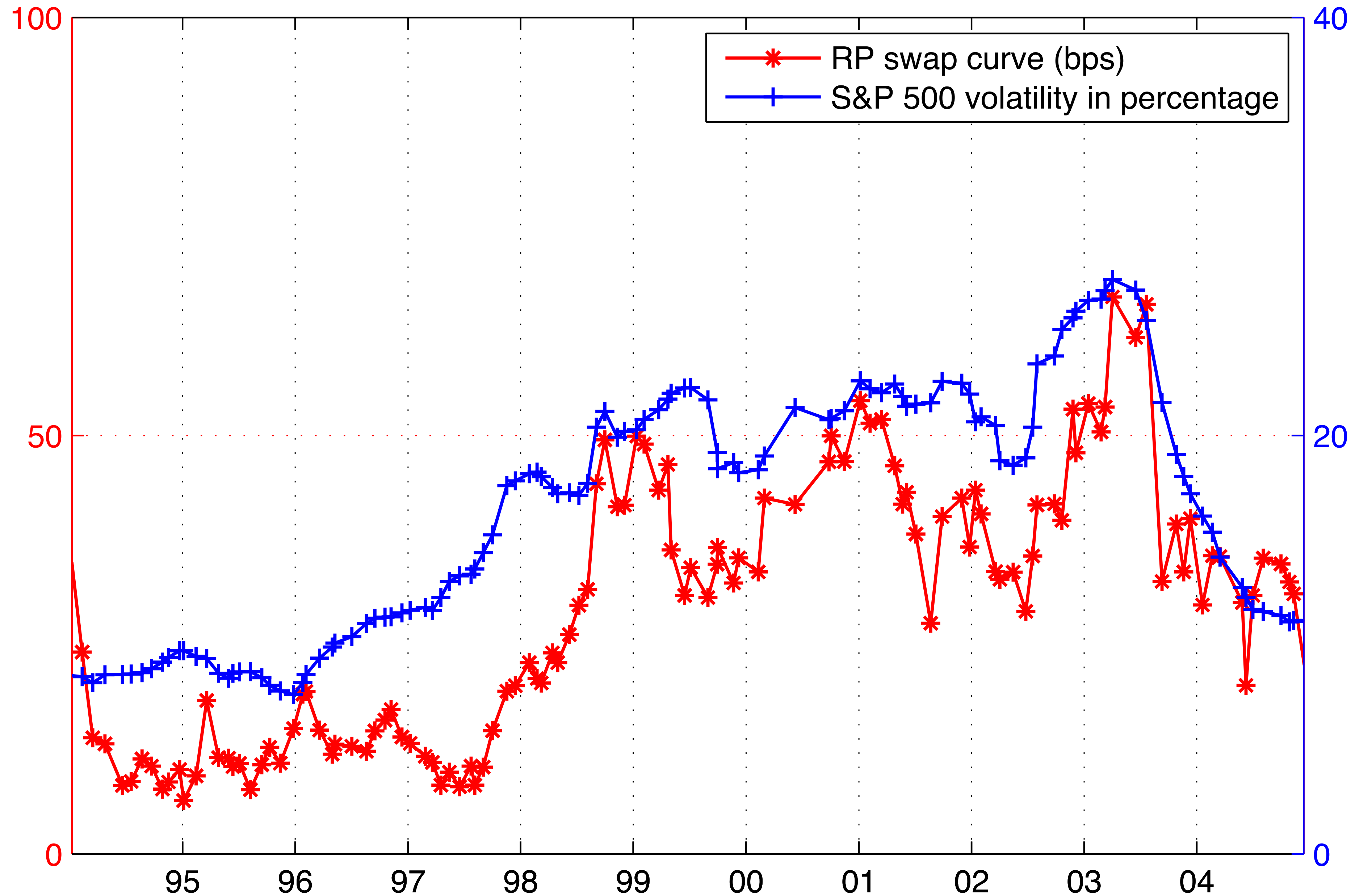
Augmenting this with a model such as Leland & Toft (1996, JF) highlights the 2nd point

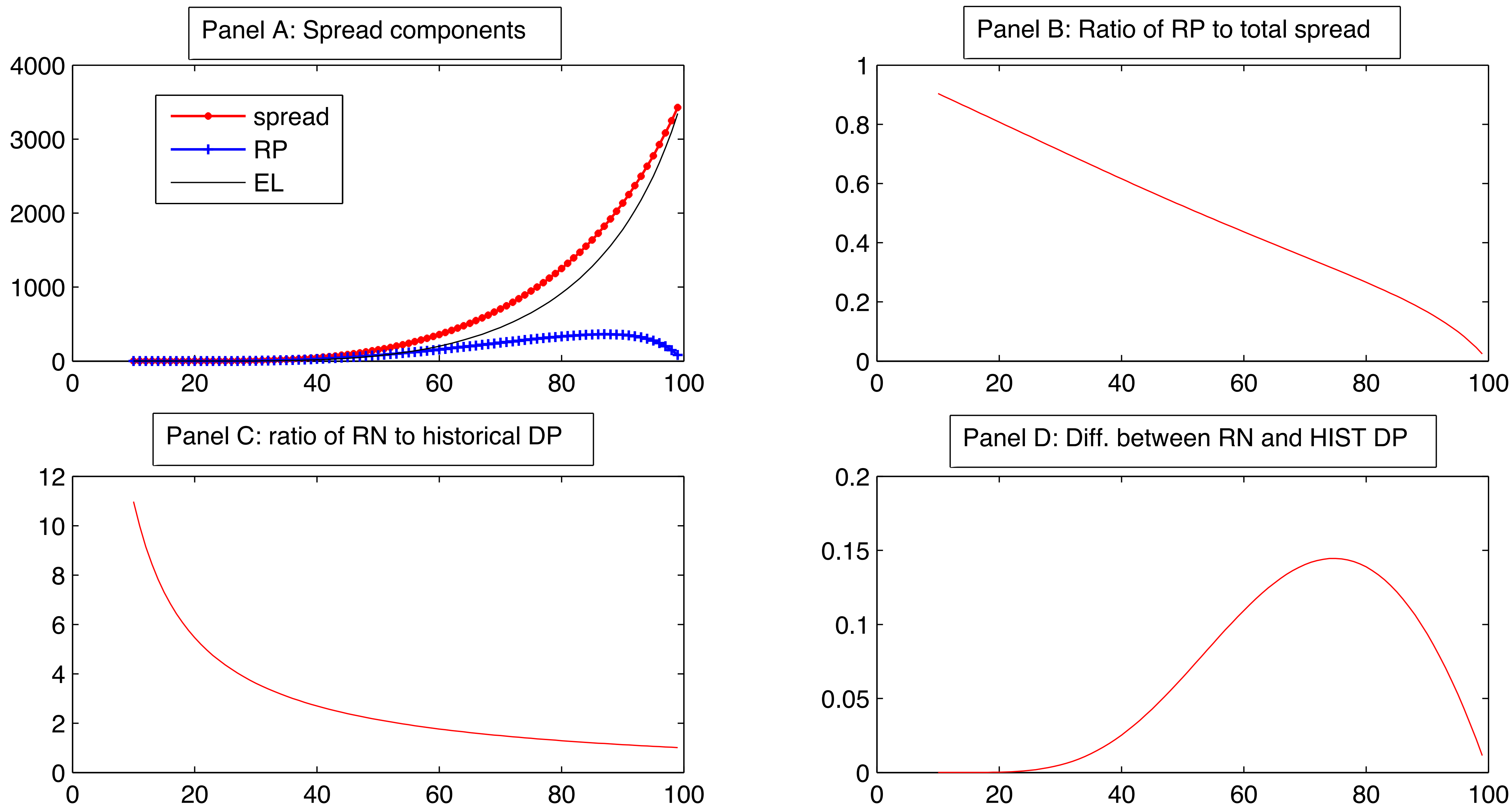
$$R_D(v; t, \sigma, N, T, r) = R_v + (R_v - R_E(v; t, \sigma, N, T, r)) \cdot \frac{E(v; t, \sigma, N, T, r)}{D(v; t, \sigma, N, T, r)}$$

What drives expected losses?



What drives risk premia?





Panel A: Spread components
spread
RP
EL
4000
3000
2000
1000
0
0
20
40
60
80
100
Panel B: Ratio of RP to total spread
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
Panel C: ratio of RN to historical DP
12
10
8
6
4
2
0
0
20
40
60
80
100
Panel D: Diff. between RN and HIST DP
0.2
0.15
0.1
0.05
0
0
20
40
60
80
100

	All		1st quartile		2nd quartile		3rd quartile		4th quartile	
Leverage	4.901	4.431	1.839	1.296	3.402	2.850	4.261	3.690	7.166	7.053
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Historical equity vol.	0.778	0.715	1.349	1.112	0 .995	.822	0 .674	0.638	1.004	0.996
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Equity return	0.001	0.001	0.001	0.001	0.001	.001	0.003	0.003	0.001	0.001
	(0.000)	(0.000)	(0.010)	(0.032)	(0.03)	(0.09)	(0.000)	(0.000)	(0.107)	(0.108)
Slope	7.114	9.593	6.923	11.38	7.351	11.637	13.520	16.548	-1.848	-1.930
	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.000)	(0.715)	(0.703)
Swap-yield	-24.292	-22.616	-7.718	-7.235	-14.618	-13.440	-23.68	-21.230	-53.463	-52.742
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Risk premium		0.364		0.637		.567		0.368		0.076
		(0.000)		(0.000)		(0.000)		(0.000)		(0.000)
Constant	-76.96		-6.36	-3.29	-25.01	-20.42	-127.17	-111.14	-300.2	-300.6
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2- within	13.00%	13.54%	10.05%	12.19%	11.87%	13.32%	14.56%	14.94%	18.44%	18.50%
R^2- between	27.23%	33.72%	0.07%	10.67%	12.14%	35.49%	17.98%	29.46%	14.70%	16.74%
R^2- overall	30.9%	36.08%	8.76%	20.35%	15.60%	31.07%	20.62%	27.74%	22.40%	23.83%
N	33.626*	33.626*	8828	8828	6,605	6,605	9078	9078	6763	6763
Number of groups	**988**	**988**	**359**	**359**	**579**	**579**	**479**	**479**	**361**	**361**

Summary so far

- spread components highly time varying both in absolute and relative terms.
- the link between risk premia in equity and credit markets is non-linear and time varying.
- RP tends to be higher in a relative sense for higher grade credits and in times of relatively low default rates.
- RP help in explaining bond spread data for high rated bonds in particular.

Derivatives markets - CDS and stock options

- We now move on to a slightly different question (based on current joint work with Christian Dorion and Redouane Elkamhi)
  - so far we have discussed splitting the total bond spread into EL and RP components and shown
    - EL / RP depend on idiosyncratic / systematic risk.
- But does knowing the split of volatility in its idiosyncratic and systematic components help to explain the total spread?
- Does it help help in pricing derivatives (options, CDS) relative to the underlying?

Does knowledge of systematic risk help in pricing derivatives relative to the underlying?

- Duan & Wei (2008, RFS) have shown that it matters for options on some 30 stocks during the early nineties.

- We will now show that risk premia help in pricing CDS as well as stock options.

    - matched sample of CDS and equity options for about 130 firms in the CDX index between 2004 and 2007.

$$b_{j,k} = \frac{\beta_{j,k}^2 \sigma_{M,k}^2}{\sigma_{j,k}^2}$$ systematic risk proportion

Stock options and systematic risk

Long-Term Options							
From AAA to A-	1698	2.7942	2.9230		3.4739	3.1730	
		(12.24)	**(12.11)**		**(10.06)**	**(12.89)**	
BBB+, BBB, BBB-	2300	3.0286		2.7293	4.7051		3.9395
		(8.97)		**(5.69)**	**(10.24)**		**(10.46)**
Systematic Risk		0.0817	0.0689	0.1783	0.0271	0.0084	0.1655
		(5.12)	**(3.64)**	**(5.49)**	(1.57)	(0.89)	**(3.37)**
Equity Volatility					-0.3122	-0.3443	-0.2223
					(-7.74)	**(-7.70)**	**(-4.33)**
Firm Size					-0.9891	-0.8392	-0.9463
					(-4.83)	**(-4.54)**	**(-2.13)**
Market Leverage					-0.0043	-0.0101	-0.0301
					(-0.48)	**(-2.65)**	(-1.37)
R^2		15.78%	10.47%	28.57%	52.77%	56.28%	51.59%
Coeff > 0		88.89%	81.48%	77.78%	63.89%	51.85%	77.78%

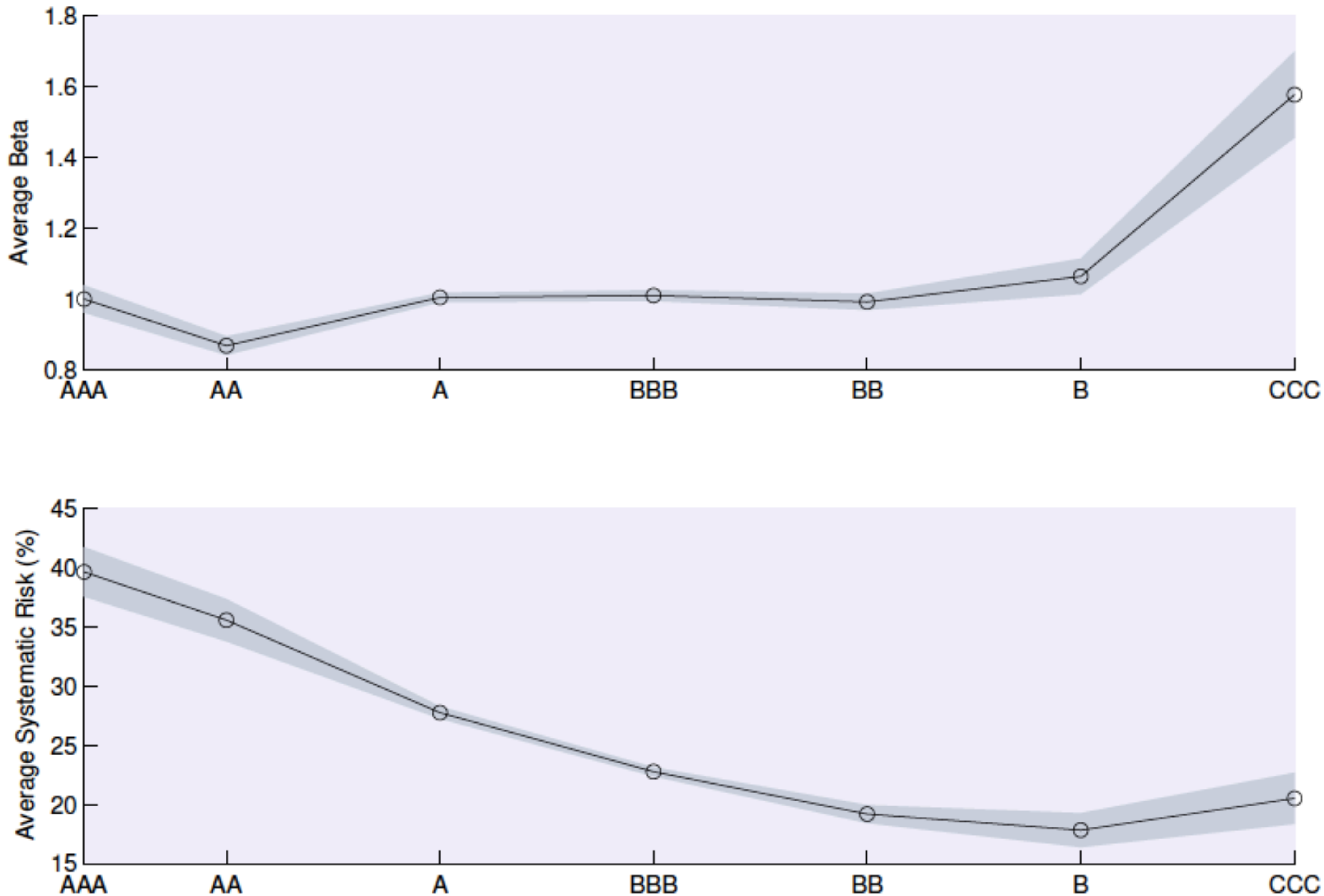
$$\frac{IV_{t,j} - \sigma_{t,j}}{IV_{t,j}} = \eta_{A,t} A_{t,j} + \eta_{B,t} BBB_{t,j} + \eta_{b,t} b_{t,j} + \eta_{c,t} \cdot Controls_{t,j} + \varepsilon_{t,j}$$

CDS spreads and systematic risk

5-years Spreads	**#Obs**			
From AAA to A-	2446	32.8290	28.3500	
		(25.36)	**(17.01)**	
BBB+, BBB, BBB-	3164	50.0600		53.2810
		(25.44)		**(30.83)**
Systematic Risk		-0.1869	-0.2395	-0.2370
		(-3.19)	**(-7.65)**	**(-3.42)**
Equity Volatility		0.9843	0.7852	1.1258
		(4.48)	**(4.08)**	**(4.37)**
Firm Size		-5.6054	-3.3371	-8.3634
		(-3.91)	**(-6.54)**	**(-3.27)**
Market Leverage		0.2128	0.2071	0.1775
		(2.75)	**(2.90)**	**(1.62)**
R^2		42.03%	34.75%	28.94%
Coeff > 0		20.37%	0.00%	25.93%

?

$$s_{t,j} = \eta_{A,t} A_{t,j} + \eta_{B,t} BBB_{t,j} + \eta_{b,t} b_{t,j} + \eta_{c,t} \cdot Controls_{t,j} + \varepsilon_{t,j}$$



Average Beta
1.8
1.6
1.4
1.2
1
0.8
AAA
AA
A
BBB
BB
B
CCC
Average Systematic Risk (%)
45
40
35
30
25
20
15
AAA
AA
A
BBB
BB
B
CCC

The RP spread

5-years Spreads	#Obs			
From AAA to A-	2446	69.8630	77.2070	
		(19.55)	**(17.43)**	
BBB+, BBB, BBB-	3164	78.2680		75.2320
		(25.54)		**(27.70)**
Systematic Risk		0.3740	0.7684	0.2475
		(2.74)	**(5.36)**	**(1.45)**
Equity Volatility		0.7352	1.6409	0.4081
		(3.29)	**(6.01)**	**(1.70)**
Firm Size		-1.0886	-7.1560	4.2976
		(-1.16)	**(-4.99)**	**(2.70)**
Market Leverage		0.4685	0.5957	0.3200
		(12.82)	**(9.19)**	**(4.15)**
R^2		16.67%	26.24%	16.84%
Coeff > 0		72.50%	92.50%	62.50%

$$\frac{s_{t,j} - EL_{t,j}}{s_{t,j}} = \eta_{A,t} A_{t,j} + \eta_{B,t} BBB_{t,j} + \eta_{b,t} b_{t,j} + \eta_{c,t} \cdot Controls_{t,j} + \varepsilon_{t,j}$$

Summary and implications.

- When pricing equity and credit derivatives relative to the underlying, systematic risk matters - prefence free pricing does not seem to hold.

- The mix between compensation for expected losses and the compensation for risk in bond and CDS spreads behaves in a complex way.

- Ratings alone do not give sufficient guidance for assessing a credit investment. Do we need risk-adjusted ratings?