

$$\min_{\sum w_i = 1} L(z) = \sum w_i B_i^T z_i \quad L = \sum x_i L_i$$

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Practical Modelling Of the Incremental Risk Charge In The Trading Book



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Joint with David Saunders

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Preface...



The current events have highlighted the need for **transparency**

- Consistent valuation and risk methodologies across asset classes
- Detailed modelling of instruments and collateral
- Counterparty credit risk
- Concentration risk and risk contributions
- Model risk
- Stress testing
- Explicit modeling of the interaction of market, credit, and liquidity risk



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Summary – IRC

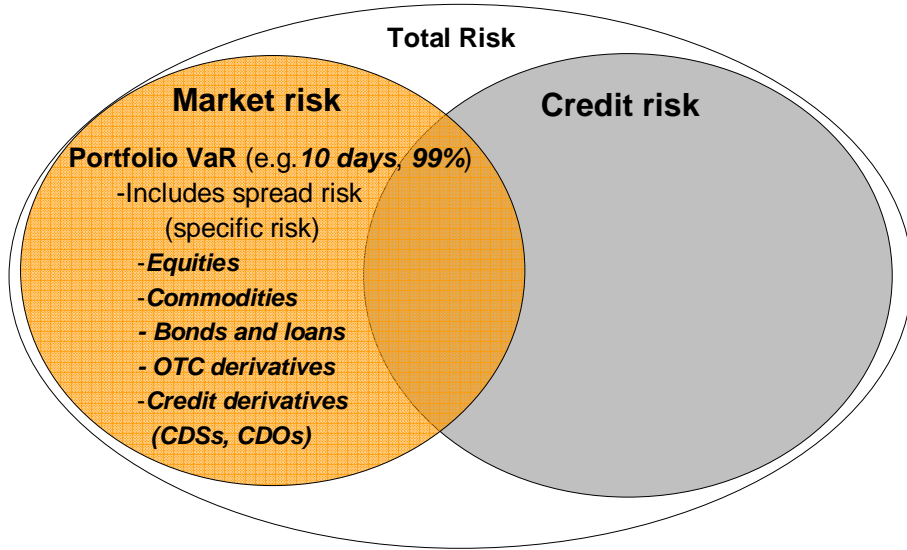
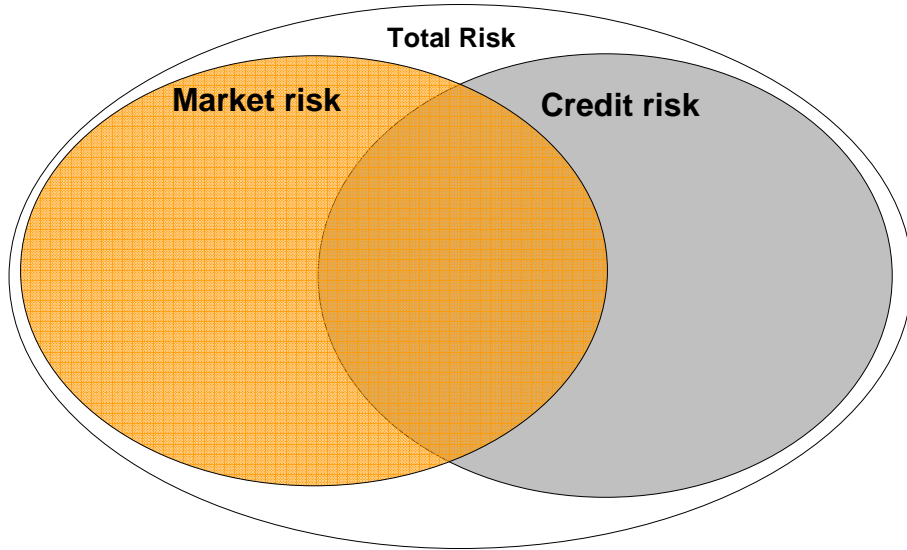


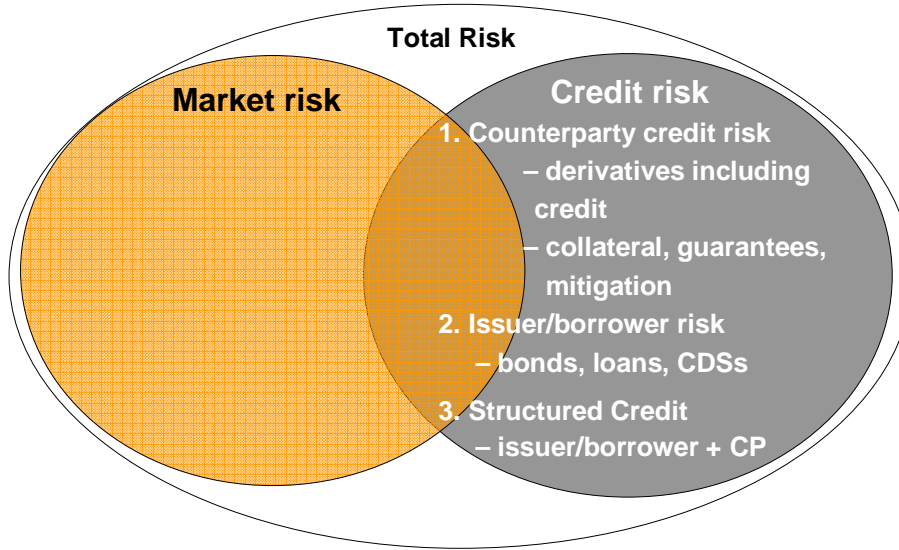
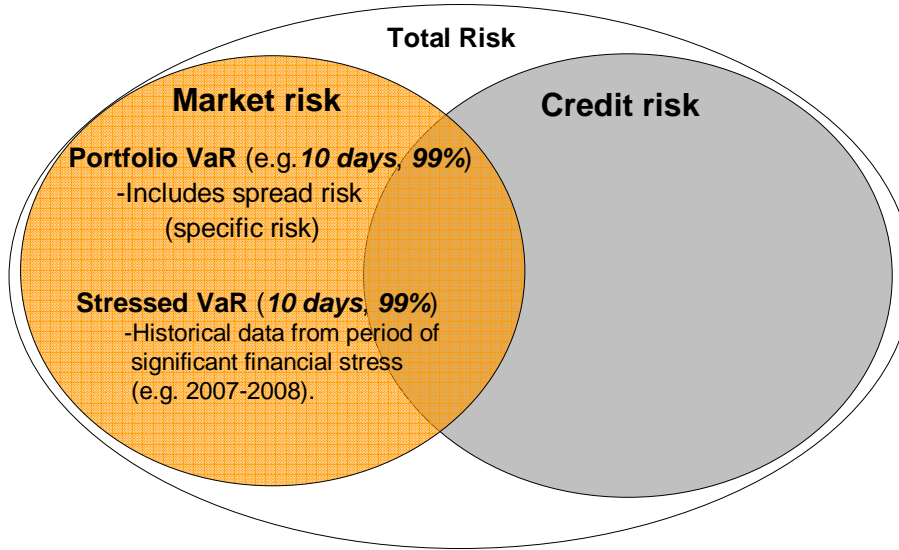
- Revisions to the Basel II market risk framework, finalized in July 2009
 - Require that banks develop a methodology for calculating a new *incremental risk charge* (IRC)
- IRC: captures credit **default and migration risks** that are incremental to the risks captured by market VaR
 - Trading book positions on unsecuritised credit products
 - *Constant level of risk* over the one-year capital horizon
 - *Liquidity horizon* – positions whose credit characteristics have improved or deteriorated are replaced so that credit characteristics are equivalent to those at the start of the liquidity horizon

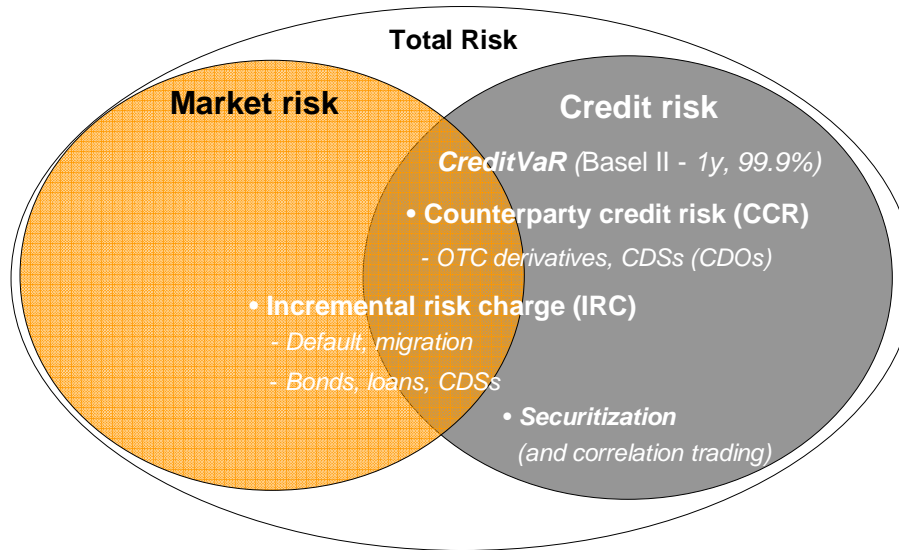
Summary – This Talk



1. Discuss the basic principles behind the IRC requirements
2. Present a robust methodology to compute IRC
 - Discuss various modelling choices, which arise in practical implementations
 - IRC methodology models the constant level of risk principle by combining
 - Repeated application of single-step credit portfolio models
 - Advanced convolution methods to model the constant risk principle
 - Advantages:
 - Transparent modelling of constant risk principle
 - In contrast to a brute-force dynamic, multi-step Monte Carlo
 - Does not need heavy parameterization or operational assumptions
 - Easy to implement – leverages existing credit portfolio tools used by banks
 - Comprehensive stress testing – understand credit risk components, risk contributions, diversification, capital benefits from long-short positions
3. Present practical examples







Market Risk (Internal Models Approach):

- General Market Risk Charge (10 days, 99% VaR):
 - Stressed Market Risk Charge (10 days, 99%)
 - Calibrated to historical data from a period of significant financial stress (e.g. 2007-2008)
- Specific Risk Charge
 - Incremental Risk Charge (IRC) – default and migration risk
 - Measured at a 99.9% confidence level, with a one-year time horizon, under the constant level of risk assumption
- Structured credit
 - Correlation trading: standardized and comprehensive treatment
 - Securitizations – outside of IRB
 - Conservative, standardized charge

Incremental Risk Charge (IRC)



- Basel II revised proposals on IRC capital in the trading book in July 2009
 - Risks: **default, credit migration** (incremental to market VaR)
- New rules come into force from at end 2010 – tight schedule
 - Banks are required to calculate the IRC at least weekly
- Basel committee expects banks to develop own internal IRC models and fall-back option is very punitive
- Measure losses at 99.9% level over a capital horizon of one year
 - Constant risk – liquidity horizons with 3m floor
- Structured credit
 - Correlation trading: standardized approach or “comprehensive treatment”
 - Securitizations – outside of IRB
 - Conservative, standardized charge

Chronology of IRC – A Non-linear Path...

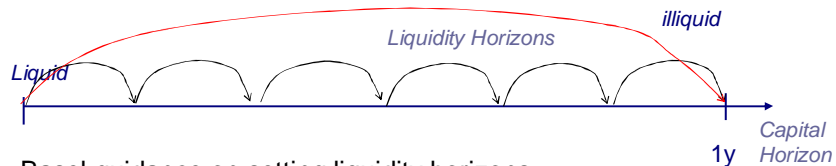


- **July 2005** – Basel Committee/IOSCO Agreement
 - Banks that model specific risk in the trading book must measure and hold capital against IDR
 - Motivated by concerns regarding illiquid credit risk-related products held in the trading book (risk not reflected in VaR)
- **October 2007** – Basel Committee IDR Guidelines Released
- **March 2008** – Expansion of Scope IDR → IRC
- **July 2008** – IRC Consultative Document
 - $IRC = F(\text{default, spread, migration, equity risks})$
- **January 2009** – Revised Consultative Document
 - $IRC = F(\text{default, migration})$
 - Securitizations removed from IRC modelling, given more conservative treatment
- **July 2009** – Final BCBS Document: Market Risk Revision
- **December 2010** – Banks have to comply with the revised requirements until the end of 2010 for portfolios and products for which they already received or applied for internal models approval for the calculation of market risk capital or specific risk model recognition before the implementation of these changes

Constant Risk and Liquidity Horizons



- **Capital horizon:** time over which credit events are measured (1-year)
- **Liquidity horizon:** frequency at which the portfolio (or transaction) is rebalanced to a target level of risk (e.g. 3m, 6m, 1y)



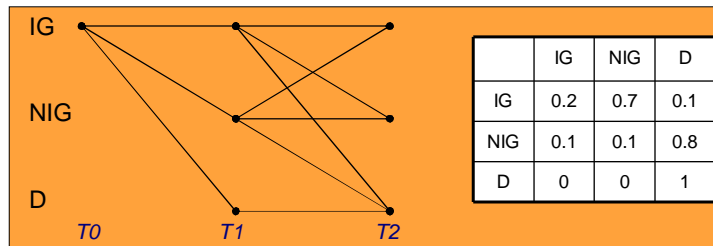
- Basel guidance on setting liquidity horizons
 - Time required to sell position or hedge all material risks *in a stressed market*
 - Sufficiently long so that selling/hedging does not materially affect prices
 - Floor of three months
 - Non-investment expected to have longer horizons than investment grade
 - Concentrated positions expected to have longer liquidity horizons

Constant Risk and Liquidity Horizon



- Constant risk assumption – liquidity shorter than capital horizon
- Estimation of liquidity horizon PDs – ratings data, KMV/market, implied from transition matrices (roots)

Example of PD effect

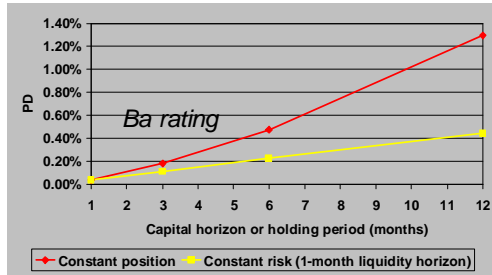


- PD (Constant Position, 2 periods) = $0.1 \times 1 + 0.7 \times 0.8 + 0.2 \times 0.1 = 0.68$
- PD (Constant Risk, 2 periods) = $0.1 \times 1 + 0.9 \times 0.1 = 1 - (1 - 0.01)^2 = 0.19$



- Constant risk assumption - scaling of PDs (scaling factor)

$$PD \text{ scaling factor} = \frac{PD \text{ with rebalancing}}{PD \text{ buy-and-hold}}$$



Rating	Moody's	MKMV
All IG	.18	.16
Ba	.34	.23
B	.60	.35
Caa-C	1.30	1.92

PD scaling factors
(1-m liquidity, 1-y capital horizon)

Source: Dunn et al. 2007



Modelling the Incremental Risk Charge

IRC Methodology and Calculation



- Underlying credit portfolio model
 - Integrated market-credit model vs. separate models and “summing” losses
- IRC Methodology – constant risk
 - Single step portfolio model: scaling-up of liquidity-horizon PDs, and transition matrices to capital horizon (constant risk assumption)
 - Multi-step MC simulation: most general but also most complex
 - Multi-step convolution method: single step to liquidity horizon and convolution (scaling-up) of loss distribution to capital horizon
- Underlying parameters and assumptions
 - Calculation of liquidity horizon transition matrices and default probabilities
 - Integrated market-credit parameters
 - Correlation effects of multi-step simulation vs. single step

Constant Risk Assumption Modelling – MC



- Issues with attempting a full Monte-Carlo simulation implementing the constant level of risk assumption
- Lack of transparency and ability to understand properties of solution
- Too many assumptions – in practice apply at instrument level (rather than portfolio level)
 - Rebalancing assumptions are crude approximations of real practice
- Correlations and multi-step models
 - By adding in more noise from different periods, multi-step models tend to produce lower default/migration correlation
 - What is the appropriate multi-step correlation structure to match a given single-step correlation structure? (which is traditionally estimated)

Constant Risk Modelling Objectives



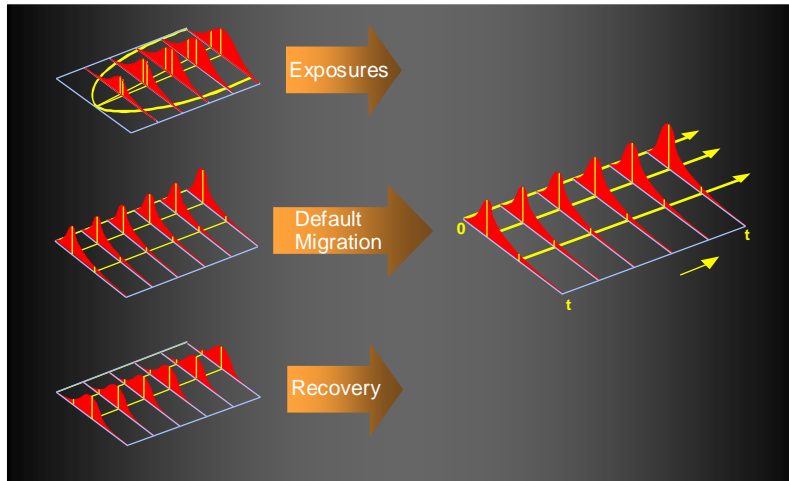
- Transparent
- Risk-sensitive
- Simple
 - Small number of meaningful parameters
 - Constant risk principle imposed at the portfolio level – rather than at each position independently
- Easy to parameterize and consistent with single-step credit portfolio parameter estimates
- Reuse existing infrastructure (single step modelling)
- Meaningful intuition for stress testing and assessing model risk
- Benchmarking

Proposed IRC Methodology



1. **Basic credit model building block:** single-step credit portfolio model
 - Losses and $EC(t)$ for a given capital horizon t
 - Loss decomposition: default vs. MtM, systematic vs. idiosyncratic
2. **Capital horizon curves:** multiple single-step credit portfolio losses
 - $t = 1m, 3m, 6m, 1y$
 - Calculation for portfolio and sub-portfolios
3. **Constant risk capital curves (liquidity horizon) – scaling**
 - Convolution (homogeneous liquidity): $EC(t) \rightarrow EC(T=1y)$
 - Alternatives: simple time-scaling: e.g. \sqrt{t} , “ad-hoc” factor
4. **Aggregation and inhomogeneous portfolios**
 - Definition of “liquidity sub-portfolios”
 - Inhomogeneous liquidity (or non-standard liquidity horizon)
 - **Multi-step convolution and copulas** for inhomogeneous horizons
 - Interpolation: weighted-averages

1. Portfolio Credit Risk Model: Components

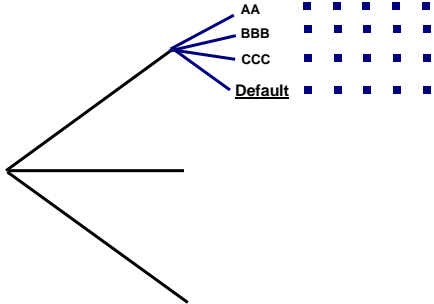


General Credit Portfolio Model

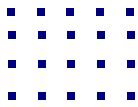


1. Scenarios:

- market factors
- credit drivers



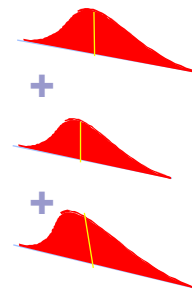
2. Conditional def./mig. probabilities



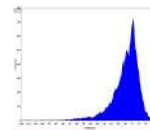
3. Conditional instrument prices & CP exposures



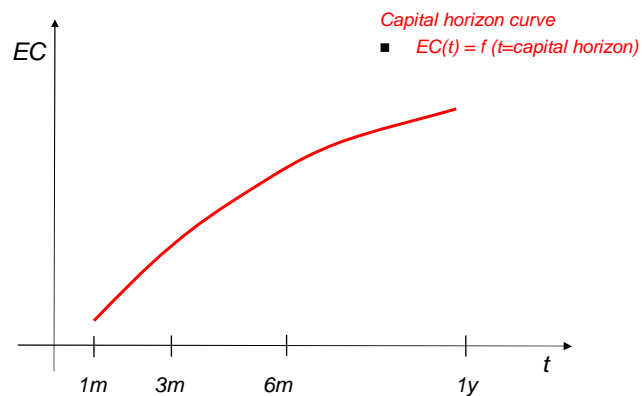
4. Conditional portfolio losses



5. Unconditional Portfolio loss distribution



1. Single-step credit portfolio model
 - Losses and $EC(t)$ for a given capital horizon t
 - Loss decomposition: default vs. MtM, systematic vs. idiosyncratic
2. **Capital horizon curve – multiple single-step credit portfolio losses**
 - $t = 1m, 3m, 6m, 1y$
 - Calculation for portfolio and sub-portfolios
 - Repeated simulation applying the single step portfolio model for different liquidity horizons
 - Scaling of default probabilities and correlations.



IRC Methodology

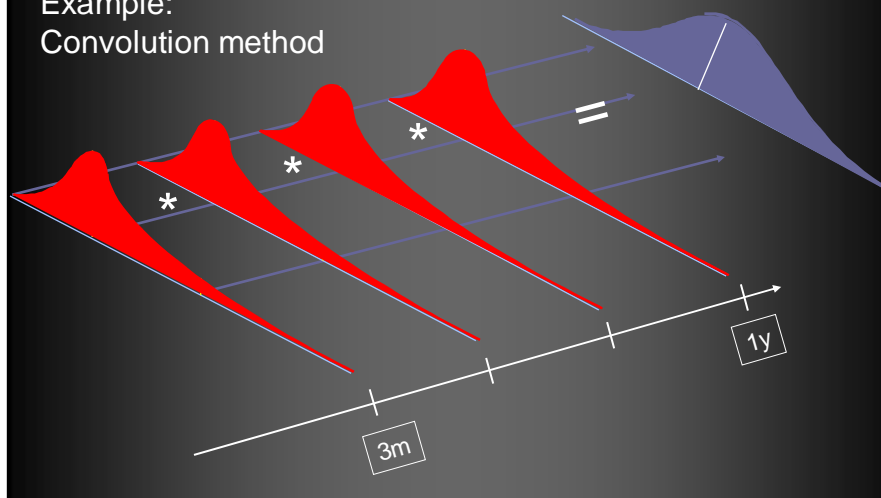


1. Single-step credit portfolio model
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 - Loss decomposition: default vs. MtM, systematic vs. idiosyncratic
2. Capital horizon curve – multiple single-step credit portfolio losses
 - $t = 1m, 3m, 6m, 1y$
 - Calculation for portfolio and sub-portfolios
3. **Constant risk capital (liquidity horizon) curve – scaling**
 - $EC(t) \rightarrow EC(T=1y)$
 - **Convolution (consecutive horizon periods are independent)**
 - **Alternatives: simple time-scaling: e.g. \sqrt{t} , “ad-hoc” factor**
 - **Calculate economic capital under the constant level of risk assumption (through simplified scaling).**
 - **Assume entire portfolio has the same liquidity horizon.**

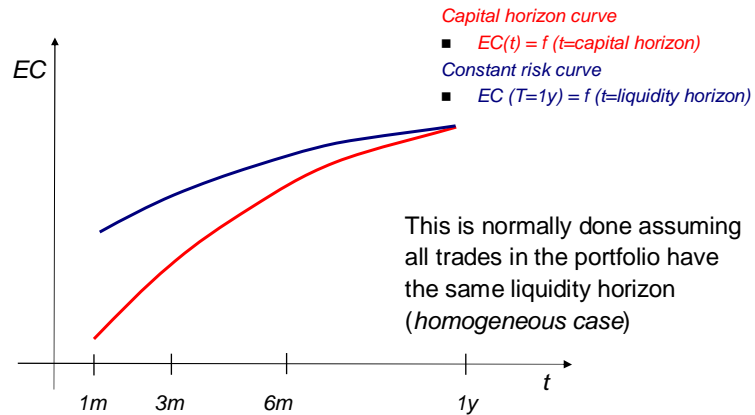
IRC – Capital and Liquidity Horizon



Example:
Convolution method



Constant Risk Curves (Liquidity Horizon)



IRC Methodology



1. Single-step credit portfolio model
 - Losses and $EC(t)$ for a given capital horizon t
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4. **Aggregation and inhomogeneous portfolios**
 - **Definition of “liquidity sub-portfolios”**
 - **Inhomogeneous liquidity (or non-standard liquidity horizon)**
 - **Multi-step inhomogeneous convolution and copulas**
 - **Interpolation: weighted-averages**

Inhomogeneous Portfolios and Interpolation



- Interpolation based on upper and lower bounds (C. Finger, 2009).
 - Example lower bounds: IRC calculated assuming homogeneous liquidity horizon of 3 months and 1 year.

$$IRC = \lambda \cdot IRC_L + (1 - \lambda) \cdot IRC_U$$

- where λ is determined by the liquidity horizons of instruments in the portfolio.
- Select the IRC corresponding to the point on the liquidity horizon curve with the 'portfolio average liquidity horizon':

$$\overline{LH} = \sum_n w_n LH_n, \quad \sum_n w_n = 1$$

- Possible choices for weights include % exposures, % expected losses, capital contributions,...

Multi-step Convolution with Copulas



- Final step can be computed using a multi-step convolution technique and copula codependence of sub-portfolio losses
 - Dynamic (multi-step) single-factor
 - Captures effectively constant risk within inhomogeneous portfolios (liquidity horizons)

■ Example:

- Portfolio $P = P1 + P2 + P4$ – sub-portfolios have liquidity 3m, 6m, 1y respectively
- Portfolio Losses $L = L1 + L2 + L4$
- Single factor credit factor Z
- Individual losses at their liquidity horizon computed using single-step credit portfolio model: $L1(t1), L2(t2), L4(t4)$
- Single factor copula model for L as follows:

$$\begin{array}{lll}
 X_1 = \Phi^{-1}(F_1(L_1(t_1))) & X_2 = \Phi^{-1}(F_2(L_2(t_2))) & X_3 = \Phi^{-1}(F_3(L_3(t_4))) \\
 r_1 = \text{Corr}(X_1, Z = Z_1) & r_2 = \text{Corr}(X_2, Z = \frac{1}{\sqrt{2}}(Z_1 + Z_2)) & r_3 = \text{Corr}(X_3, Z = \frac{1}{2} \sum_{i=1}^4 Z_i)
 \end{array}$$

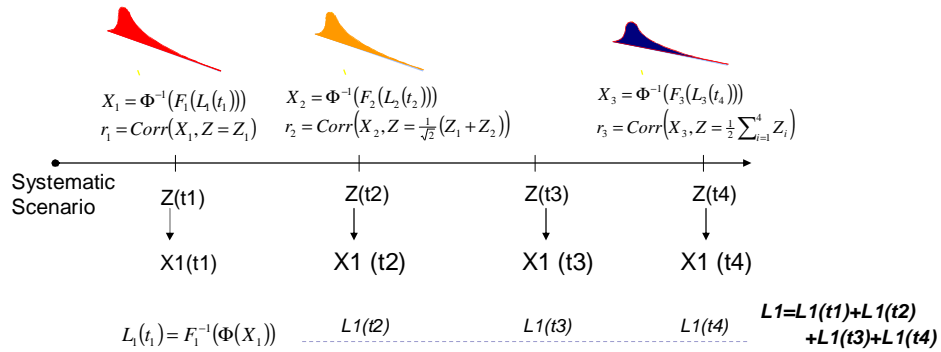
Z(t1)
Z(t2)
Z(t3)
Z(t4)

- Correlation estimated directly from single-step simulation

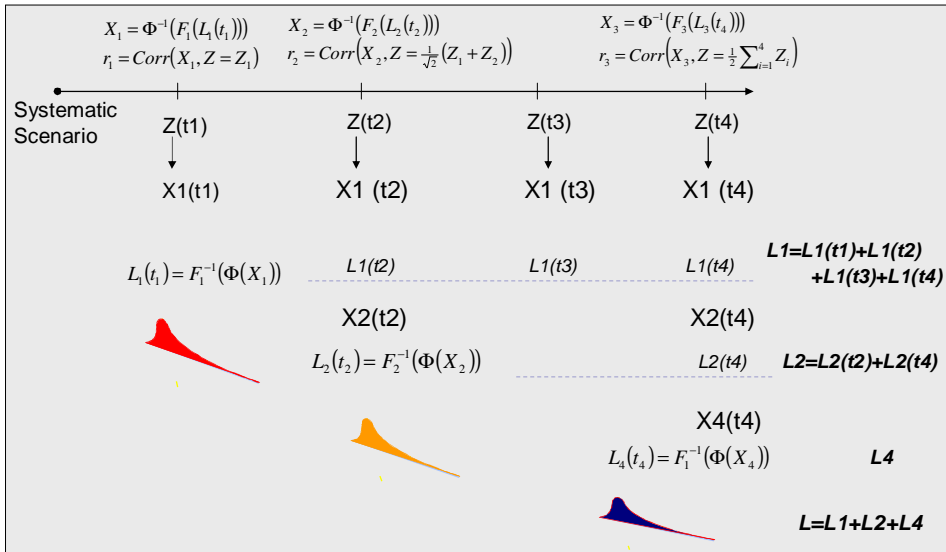
Multi-step Convolution with Copulas



- Final convolution through Monte Carlo simulation
- Each scenario: 4 time steps – 4 increments of Z
 - Can be independent, or more generally we can build auto-correlation



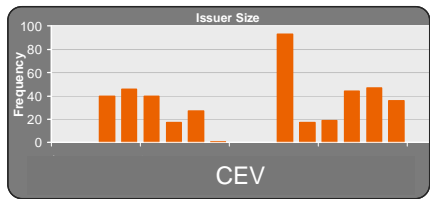
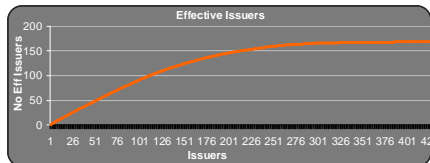
Multi-step Convolution with Copulas



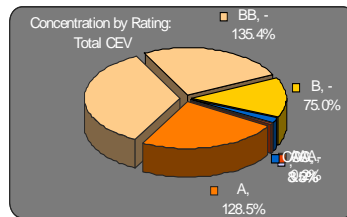
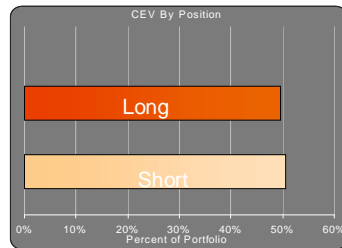
Examples

Example - Portfolio

- Portfolio: CDS and bonds, ~3,500 positions
- 425 Issuers (170 overall long issuers)
- Slightly negative cash equivalent value (CEV)



Inc.



Transition Matrix



	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.92316	0.06735	0.00717	0.00083	0.00125	0.00011	0.00002	0.00011
AA	0.00595	0.91217	0.07055	0.00825	0.00113	0.00144	0.00029	0.00023
A	0.00064	0.01796	0.91791	0.05406	0.00593	0.00223	0.00034	0.00092
BBB	0.00022	0.00195	0.03810	0.90378	0.04208	0.00919	0.00141	0.00328
BB	0.00040	0.00080	0.00394	0.05178	0.84643	0.07464	0.00767	0.01435
B	0.00002	0.00074	0.00225	0.00457	0.05127	0.84682	0.03566	0.05868
CCC	0.00000	0.00008	0.00327	0.00514	0.01282	0.08201	0.64225	0.25443
D	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

1 year

3 months

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.98017	0.01795	0.00139	0.00013	0.00033	0.00001	0.00000	0.00003
AA	0.00158	0.97709	0.01883	0.00179	0.00023	0.00036	0.00008	0.00003
A	0.00016	0.00479	0.97845	0.01447	0.00136	0.00051	0.00008	0.00019
BBB	0.00005	0.00044	0.01019	0.97454	0.01157	0.00214	0.00035	0.00071
BB	0.00011	0.00019	0.00083	0.01427	0.95842	0.02105	0.00201	0.00311
B	0.00000	0.00019	0.00056	0.00091	0.01447	0.95838	0.01114	0.01435
CCC	0.00000	0.00001	0.00094	0.00143	0.00338	0.02560	0.89470	0.07395
D	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

Example – Conditional Loss Matrices

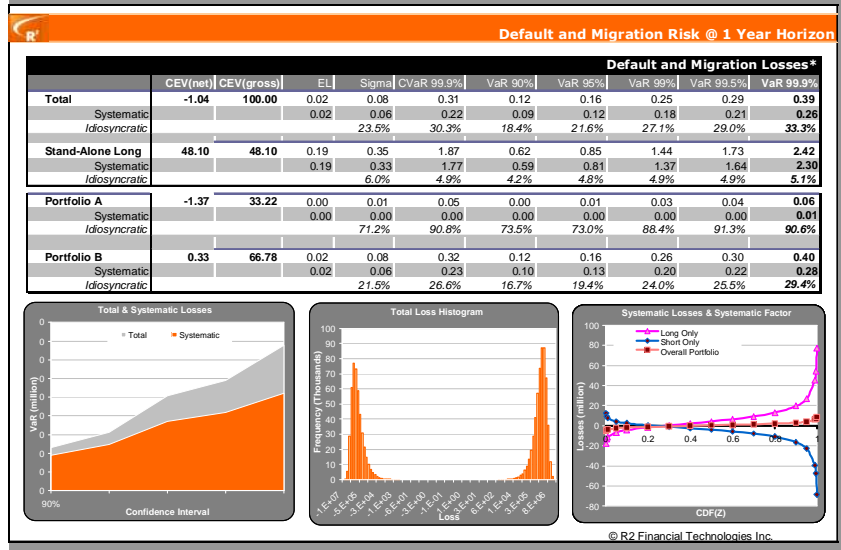


Exposure Summary										
Loss at 1Y Horizon*										
	Initial State	Issuers	Credit Scenario							
			AAA	AA	A	BBB	BB	B	CCC	D
Long	AAA	0	-	-	-	-	-	-	-	-
	AA	12	0.0	0.0	-0.1	0.4	5.1	6.4	23.6	39.9
	A	21	-0.2	-0.1	0.0	1.0	6.0	8.0	25.6	40.5
	BBB	53	-2.5	-2.2	-1.5	0.0	1.5	10.4	32.8	41.7
	BB	52	-8.4	-8.0	-6.6	-3.7	0.0	10.8	35.4	39.2
	B	28	-33.7	-32.8	-29.8	-25.2	-23.7	0.0	37.4	30.0
Short	CCC	9	-93.7	-92.4	-88.0	-81.4	-76.1	-48.5	0.0	-3.5
	AAA	2	0.0	0.0	0.0	0.0	-1.0	-5.1	-13.2	-40.3
	AA	14	0.2	0.0	-0.9	-2.7	-4.1	-8.8	-14.1	-33.5
	A	64	1.0	0.8	0.0	-1.4	-2.5	-7.9	-14.5	-32.0
	BBB	122	3.1	2.7	1.7	0.0	-0.2	-7.9	-15.6	-33.0
	BB	39	1.9	1.5	1.1	0.3	0.0	-7.0	-14.2	-32.3
Overall	B	7	1.4	1.4	1.3	1.3	0.8	0.0	-0.9	-20.4
	CCC	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-14.7
	AAA	2	0.0	0.0	0.0	0.0	-1.0	-5.1	-13.2	-40.3
	AA	26	7.6	0.0	-30.9	-67.0	25.7	-75.8	254.5	148.6
	A	85	2.0	1.6	0.0	-1.8	1.5	-7.7	-1.6	-22.1
	BBB	175	4.3	3.6	1.9	0.0	2.1	-3.4	15.2	-17.2
BB	91	-13.0	-12.4	-10.5	-6.0	0.0	13.4	50.2	44.0	
B	35	-37.5	-36.5	-33.1	-28.0	-26.4	0.0	41.7	31.1	
CCC	13	-244.4	-241.0	-229.4	-212.3	-198.5	-126.5	0.0	-32.8	

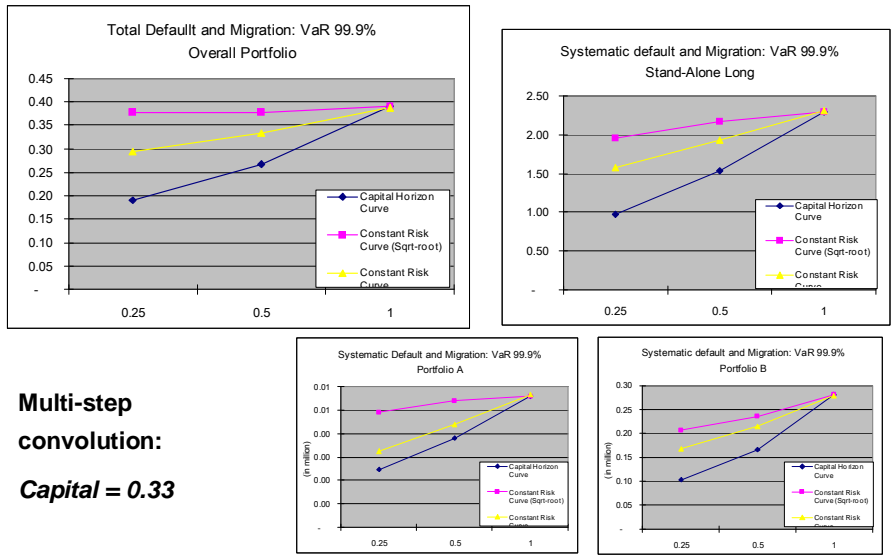
* % of CEV (t₀)

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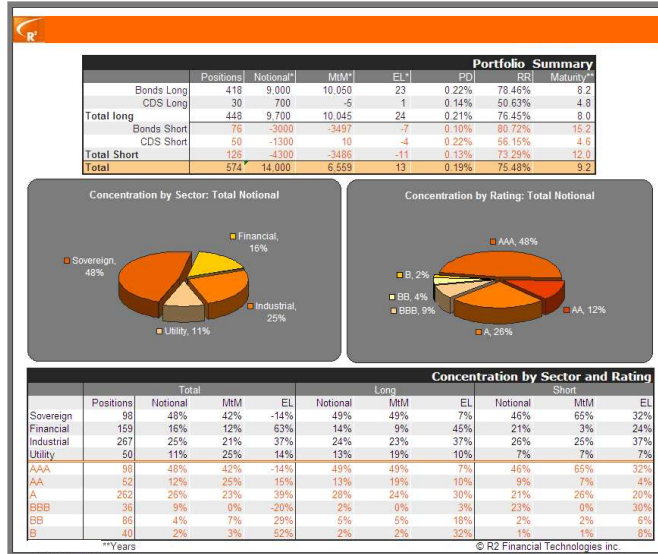
Example – Constant Portfolio Capital



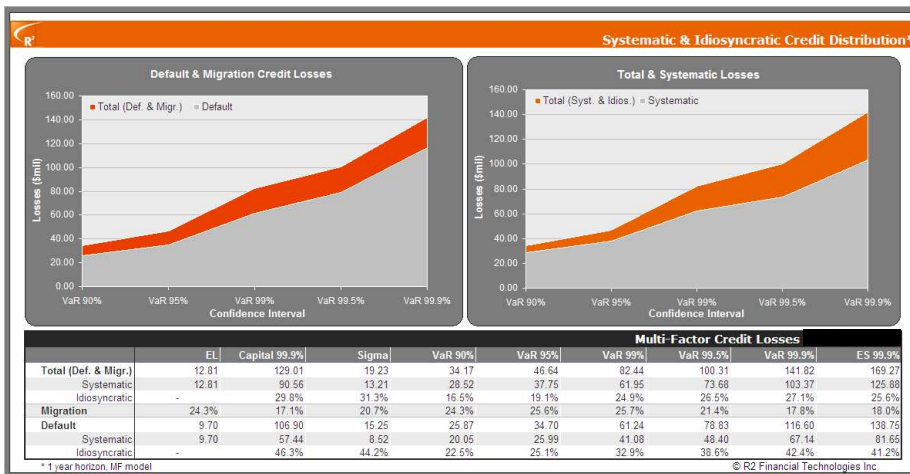
Example – Capital Horizon and Constant Risk Curves



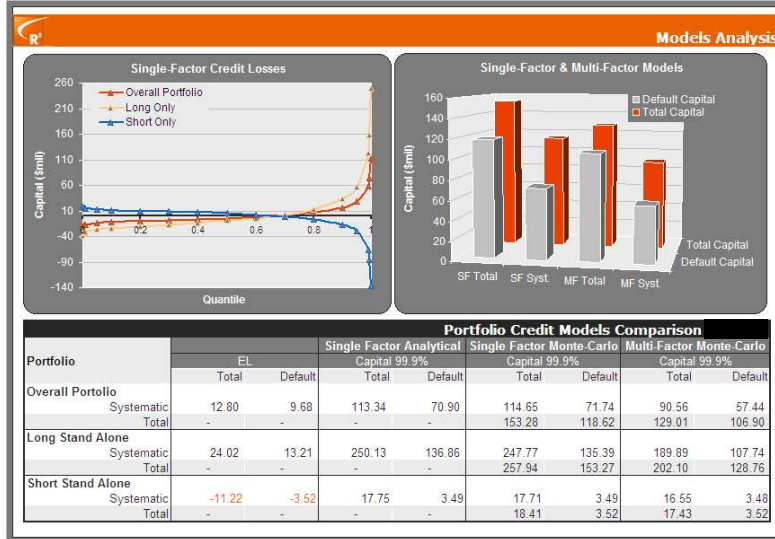
Example 2 – Credit Portfolio Modelling (long-short)



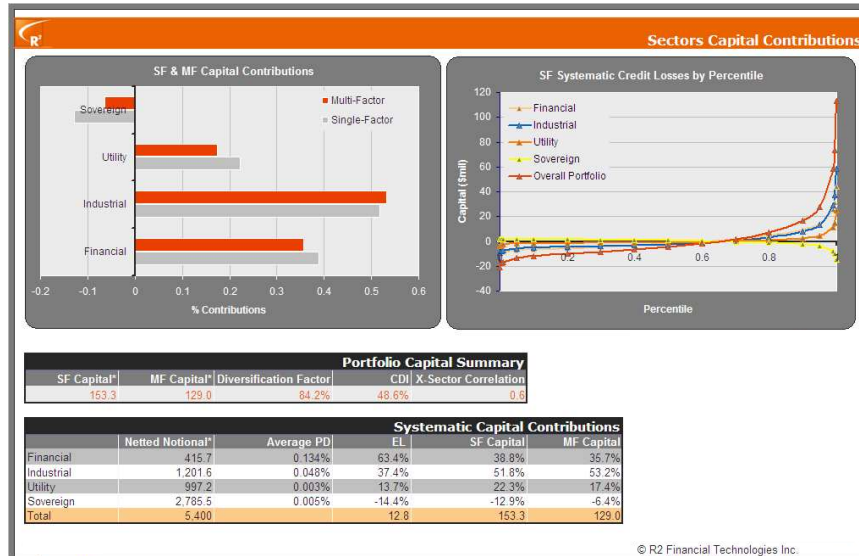
Multi-Factor Credit Losses



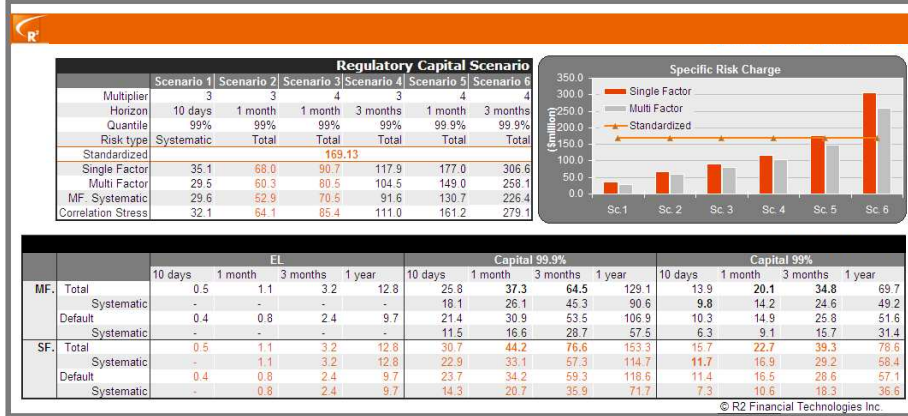
Credit Models Comparison



Loss Decomposition and Diversification



Capital Stress Tests



Concluding Remarks



Summary – This Talk



- Present a robust methodology to compute IRC
- Discuss various modelling choices, which arise in practical implementations
- IRC methodology models the constant level of risk principle by combining
 - Repeated application of single-step credit portfolio models
 - Advanced convolution methods to model the constant risk principle
- Advantages:
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 - In contrast to a brute-force dynamic, multi-step Monte Carlo
 - Does not need heavy parameterization or operational assumptions
 - Easy to implement – leverages existing credit portfolio tools used by banks
 - Comprehensive stress testing – understand credit risk components, risk contributions, diversification, capital benefits from long-short positions

$$\min_{\text{risk}} L(z) \quad \text{s.t.} \quad \sum z_i^2 = C \quad \sum w_i B_i^* z_i \quad L = \sum x_i L_i$$

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