Hedge Funds as credit derivatives

Joint work with

Marcos Escobar, Ryerson University

Stefan Kraemer, Florian Scheibl, Rudi Zagst, T.U. Munich.

A long-short pair trade

- The fund has $1000. The manager is going to purchase stock 9 units of stock A, and sell-short 9 units of stock B. Both are valued at $100 each. After a year, A is worth $110, B is $105.

Assets at Prime Broker

(Before trade)

•$1000

Assets at Prime Broker

(After trade)

- $1000
- -$900 + 9 A
- +$900 – 9 B

Assets at Prime Broker

(After one year)

- $1030
- 990
- -945
- -9

$ 1094

Leveraged version

- Assumptions: 50% collateral for long trades, 80% collateral for short trades.

Securities at Prime Broker

- 9 A ($900):
- – 9 B (-$900):

Collateral required:

$450+$720=$1170

Cash from short sale: $900

Cash required: $270

Securities at Prime Broker

- 9 A ($990):
- – 9 B (-$945):

Profit: $75

ROR: 28%

Skew and kurtosis

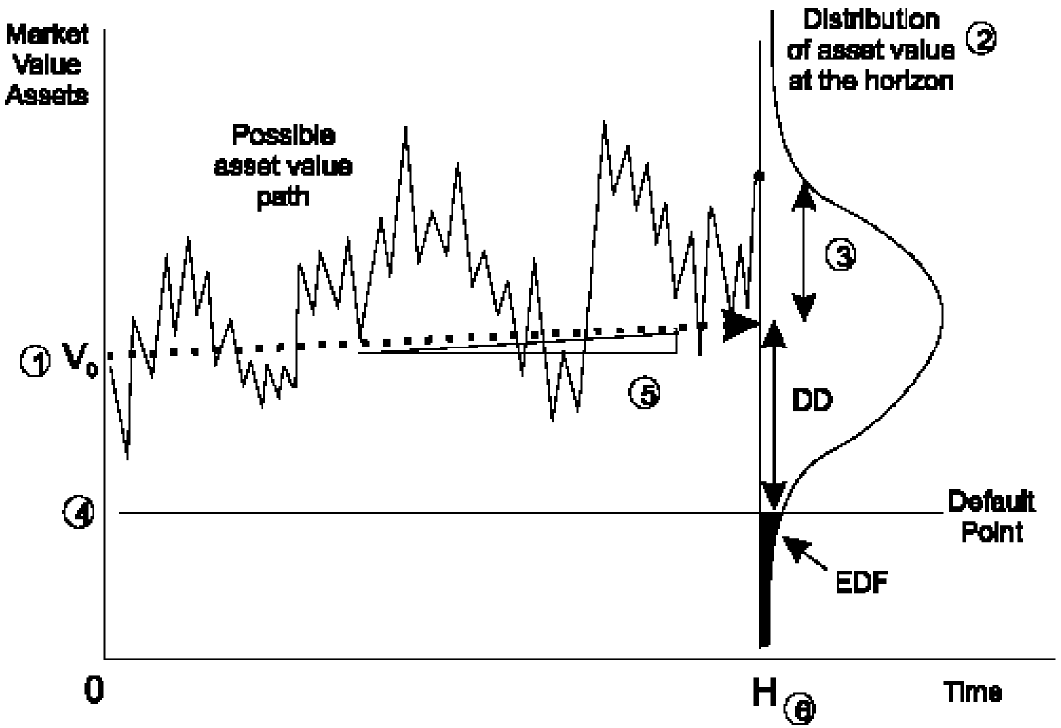
- Skew is a measure of asymmetry. It is the normalized third moment.

$$s = \frac{1}{\sigma^3} \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left(r_i - \bar{r} \right)^3$$

- Kurtosis is a measure of spread. It is the fourth moment, minus 3.
  Platykurtotic: k<0
  Leptokurtotic: k>0
  Mesokurtotic: k=0.

$$\kappa = \frac{n(n+1)}{\sigma^4 (n-1)(n-2)(n-3)} \sum_{i=1}^{n} \left(r_i - \bar{r} \right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)}$$

The Merton model of default



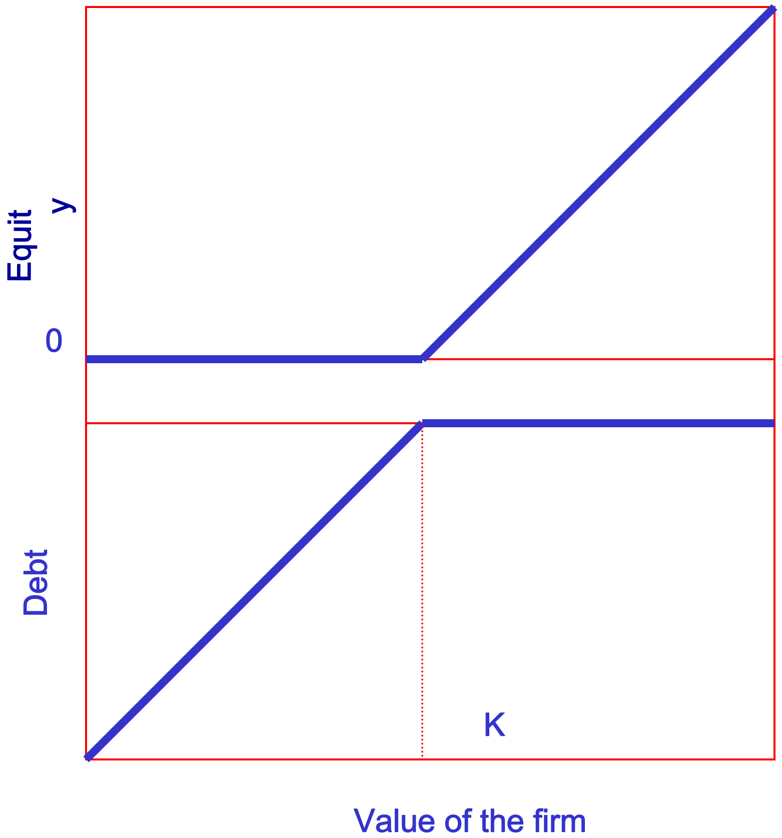
A simple setting

In its simplest situation, assume

- The firm's value equal to V.
- The firm issued a zero-coupon bond due in one time unit equal to K.

If the firm's value is higher than K, the bondholders get their bond payment, shareholders get the excess assets.

If the value of the firm is less than K, the bondholders get the value of the firm V, and equity value is 0. The firm would then be in default.



Equity values and option prices

In our simple example before, stock value at expiration is

$$S_T = \mathrm{Max}(V_T - K, 0)$$

Since the firm's value equals equity plus bonds, we have that the value of the bond is

$$B_T = V_T - \mathrm{Max}(V_T - K, 0)$$
$$= \mathrm{Min}(V_T, K)$$

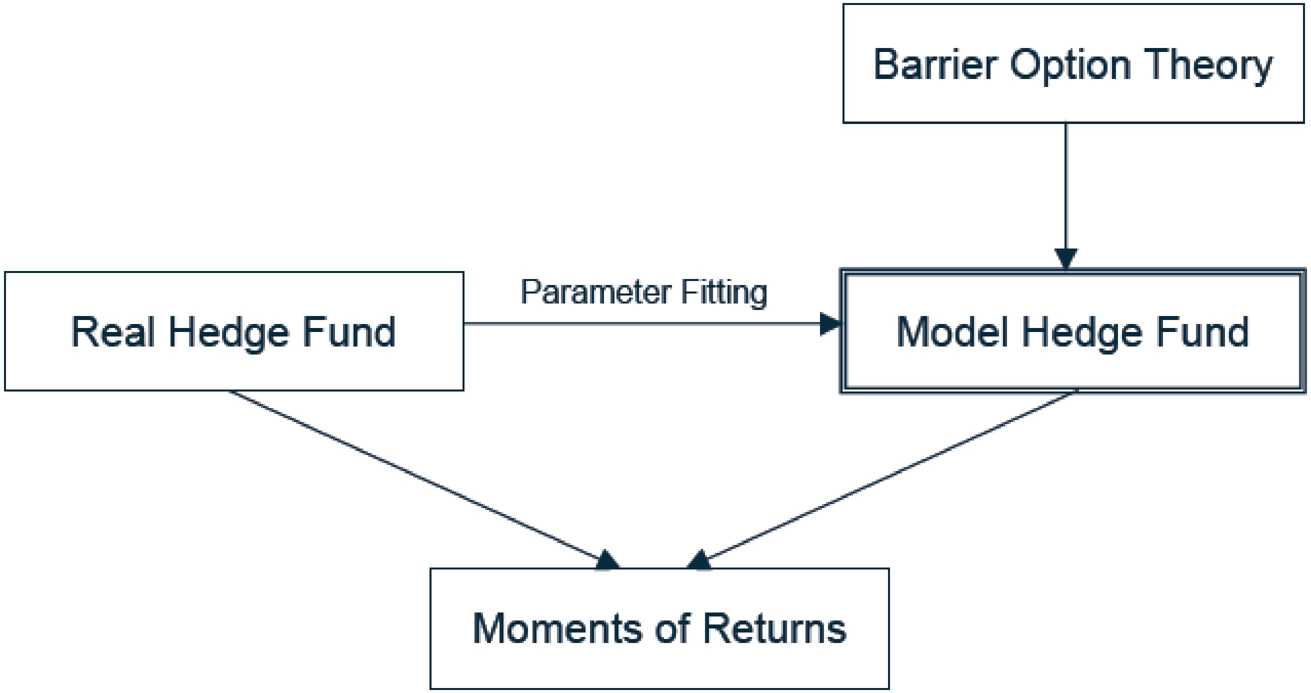
Improved default models

- Black–Cox.
  - Default can occur before expiration of the liabilities.
  - Equity is a barrier option on the assets.

$$S_t = E\left[(V_T - D_T)^+ \cdot 1_{\min V(s) > H}\right]$$ where 0<s<T

- Hedge fund default models:
  - If the skew is positive, assets are random and liabilities is fixed. The fund is a long barrier option.
  - If the skew is negative, assets are fixed and liabilities are random. The fund is a short barrier option.

A hedge fund model



Model landmarks

- Model parameters (asset volatility, leverage) are obtained from hedge fund return data.
  - Barrier option price has an explicit formula when skew is positive.
    - Constant Barrier
    - Random strike price.

$$S_t := e^{r(T-t)} \int_{\ln\frac{H}{V_t}}^{\infty} \int_{-\infty}^{x_2+\ln\frac{V_t}{D_t}} \left(V_t e^{x_2} - D_t e^{x_1}\right) P\left(X_2(T-t) \in dx_2, X_1(T-t) \in dx_1, m_{T-t}^{X_2} > \ln\frac{H}{V_t} \right) dx_1 dx_2$$

  - Calibration is obtained by the method of moments: skewness, kurtosis and autocorrelation.
- Future hedge fund returns are obtained by simulation.

Applications

- Portfolio construction
- Hedge fund classification