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Valuation and Risk of Structured Credit Products and Bespoke CDOs: *A Scenario Framework*

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Preface – in the News…

Last week WSJ

- *"**Credit Suisse** cut the value of some asset-backed securities by **$2.85 billion**, blaming some of the hit on "**mismarkings and pricing errors"** by traders…"*

- ***Now, Lehman Gets Pelted.** Many investors have been surprised at the ability of Lehman Brothers Holdings Inc. to navigate the credit crunch, given the size of its exposure to potential land mines. But… Lehman is sitting on a big pile of commercial real-estate loans, and… credit markets have worsened… now facing a **write-down in the $1.3 billion** range…"*

Preface – in the News…

- *"**Subprime crisis worse than 1998, says S&P**"*
- *"**Bifurcation or Meltdown…** Since BS spectacularly announced an initial $3.2B rescue of BSAM HG structured credit fund (June 22), funds…have seem to topple like dominoes…"*
- *"Edward Cahill, head of European CDOs at BarCap has resigned…decision comes after… **two structures downgraded by S&P… by 16 notches**, with the most senior notes going **from AAA to CCC+**"*

Sources: WSJ, Reuters, Risk, Banking Technology,, Dow Jones, FT, Bloomberg

Preface – in the News…

- *"It all happened exactly like he said it would happen... In every single detail…"*
  - *"The hedge fund creator's name was **John Paulson**… by **making between $3 billion and $4 billion** for himself in 2007, he appears to have set a Wall Street record… **no one has ever made so much so fast**."*
- Investors blaming a *"1 in 10,000 years" event…*

Sources: Reuters, Risk, Banking Technology,, Dow Jones, FT,, G&M, Bloomberg

Motivation for the Paper

1. Valuing bespoke CDOs - general view and standard practice:
   - Parameters of model derived to match quoted index tranche prices
   - Model directly applicable for non-standard tranches on the same basket (or very similar basket)
   - For bespoke portfolios, common practice: Mapping procedure
     - Requires some judgment and is “inevitably somewhat ad hoc"
2. *Cash CDO* (ABS, loans) = *bespoke portfolio* + *complex cashflow waterfall*
   - Other risks in addition to default – prepayment
   - Difficult to model and computationally intensive
   - Common practice for valuation and analysis
     - Reliance on ratings as determinants of prices and risk
     - Single-scenario valuation (lack of consistent “correlation” view)
3. Generally Lack of integrated view of synthetic and cash products and single-name credit derivatives: pricing and risk management

Summary

We introduce a general “scenario framework” – valuation and risk profiling

- *Implied factor distributions* and *weighted MC* techniques
  - Multi-factor credit models
    - Consistently characterize concentrations/correlations for different baskets
  - Weighted Monte Carlo techniques (used in options pricing)
  - CDO analytics and computational techniques
- Structured finance CDOs
  - Flexibly incorporate cash-flow and waterfall engines
  - ABS and CLO products – effectively apply prepayment and LGD models
- Basic idea: set of scenarios where instruments can be consistently valued
  - Imply “risk-neutral” joint distribution (process) for *underlying systematic risk factors*
    - Observed (liquid) prices (e.g. CDSs, index tranches)
    - Prior or “quality” preferences on distribution; subjective views
- This paper focuses on a static version, but methodology is general and can be applied to a dynamic setting

PART 1

Introduction: Valuing CDOs

Synthetic CDO

- Underlying pool of credit default swaps – divided into "tranches"

Tranche	Attachment	Detachment
Equity	0%	3%
1st Mezzanine	3%	7%
2nd Mezzanine	7%	10%
Senior	10%	15%
Super Senior	15%	30%

Size of the *n*-th tranche

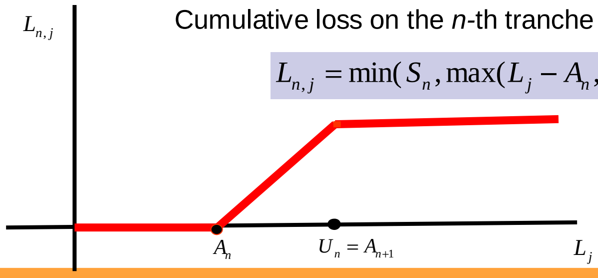
$$S_n = U_n N - A_n N = N \cdot (U_n - A_n)$$

Cumulative portfolio loss of up to t_j

$$L_j = \sum_k N_k \cdot LGD_k \cdot 1_{\tau_k \le t_j}$$

Cumulative loss on the *n*-th tranche up to time *tj*

$$L_{n,j} = \min(S_n, \max(L_j - A_n, 0))$$



Background – Pricing Synthetic CDOs

- Standard model for pricing synthetic CDOs: single-factor Gaussian copula (Li 2001)
  - Codependence through a one-factor Gaussian copula of *times to default*
  - Single parameter to estimate (correlation for all obligors in portfolio)
- Basic model does not simultaneously match market prices of all traded tranches
  - "Correlation skew" – set of correlations that match the prices of all tranches
- Base correlations – alternative to tranche correlations
  - Implied correlations of equity tranches with different attachment points (mezzanine/senior tranches as difference between two equity tranches)
- Interpolation (or extrapolation) model
  - Calibrated to observed tranche prices (e.g iTraxx or CDX)
  - Pricing of bespoke portfolios – mapping (risk of bespoke vs. index portfolio)

Single-Factor Gaussian Copula (Times to Default)

$$L_T = \sum_k 1_{\tau_k \le T} \cdot V_k(\tau_k) \cdot LGD_k(\tau_k)$$

- Cumulative default time distribution functions

$$F_k(t) = \Pr(\tau_k \le t)$$

- Creditworthiness index
  - Z systematic factor (Gaussian)

$$Y_k = \sqrt{\rho_k}\, Z + \sqrt{1-\rho_k}\, \varepsilon_k$$

- Default times
  - Mapping to Gaussian

$$Y_k \le \Phi^{-1}(F_k(t)) \iff \tau_k \le t$$

$$\tau_k = F_k^{-1}(\Phi(Y_k))$$

- Conditional on Z, default times are independent

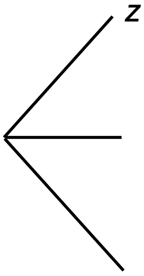
$$p_k^Z(t) = \Pr(\tau_k \le t | Z) \qquad q_j^Z(t) = \Pr(\tau_k > t | Z)$$

- Explicit formulae for conditional default probabilities

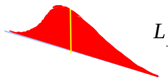
$$p_k^Z(t) = \Phi\left(\frac{\Phi^{-1}(F_k(t)) - \sqrt{\rho_k}\, Z}{\sqrt{1-\rho_k}}\right)$$

General Framework: Gaussian Copula Model

1. Scenarios: systematic factors
2. Conditional def. prob.
3. Conditional portfolio losses (discounted)



$$p_k^Z(t)=\Phi\left(\frac{\Phi^{-1}(F_k(t))-\sqrt{\rho_k}\,Z}{\sqrt{1-\rho_k}}\right)$$



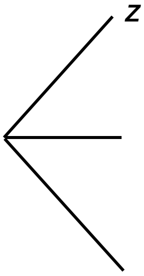
$$L_j(Z)=\sum_k N_k\cdot LGD_k(Z)\cdot 1_{\tau_k\le t_j}(Z)$$

- Conditionally independent obligor losses → convolution methods:
  - Recursions (Andersen et al, Hull-White)
  - Full simulation (independent Bernoulli variables)
  - LLN, or CLT approximation
  - Poisson approximation

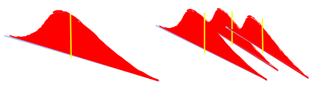
General Framework: Gaussian Copula Model

1. Scenarios: systematic factors
2. Conditional def. prob.
3. Conditional portfolio losses (discounted)

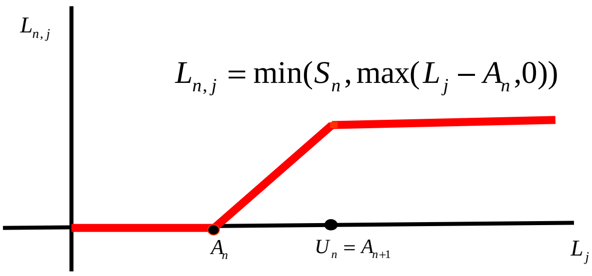
Conditional tranche losses



$$p_k^Z(t)=\Phi\left(\frac{\Phi^{-1}(F_k(t))-\sqrt{\rho_k}\,Z}{\sqrt{1-\rho_k}}\right)$$



$$L_{n,j}=\min(S_n,\max(L_j-A_n,0))$$



General Framework: Gaussian Copula Model

1. Scenarios: systematic factors

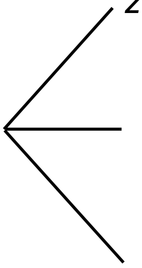
2. Conditional def. prob.

3. Conditional portfolio losses (discounted)

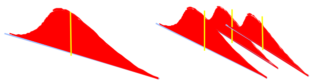
Conditional tranche losses

Conditional value of tranches

Z



$$p_k^Z(t) = \Phi\left(\frac{\Phi^{-1}(F_k(t)) - \sqrt{\rho_j}\, Z}{\sqrt{1-\rho_j}}\right)$$



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Calibration:

- <u>Correlation of each tranche *j*</u>
- *Base correlation*
  - *Equity tranches*

4. Expected tranche losses & values

$$\sum_{\omega} p_\omega E[V \mid Z_\omega]$$

Alternative Models – Background

- Alternative Copula methods:
  - NIG, *t*, double-*t*,, Clayton, Marshall-Olkin copulas (Burtschell et al 2005)
  - Stochastic correlations (e.g. Gaussian mixtures, Li 2005)
  - Local and marginal compound correlations (Laurent 2005)
  - Alternative default intensity processes: e.g. Intensity Gamma model (Joshi & Stacy 2006)
- *Implied* loss distribution or hazard rates approaches (non-parametric)
  - Hull-White 2006, Walker 2006, Brigo et al 2006
    - Generally homogeneous models (with some extensions)
- Dynamic models (generally through Monte Carlo methods)
  - Reduced form – dependent default intensities (Duffie & Garleanu 2001)
  - Structural (Merton type) – multi-step default boundary (Hull et al 2005)
  - Dynamic loss distribution processes (Giesecke, Cont, Schonbucher 2005, SPA 2005)
  - Implied processes: Hull-White (2007), Walker (2007)

Bespoke Portfolios and Mappings

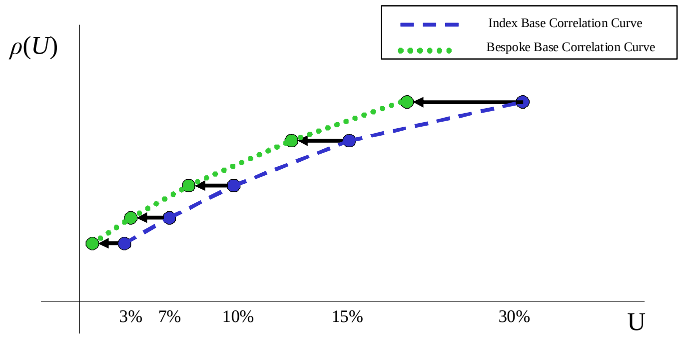
- Main idea: base correlations correspond to different levels of risk in the reference portfolio
  - By finding the same risk levels on the bespoke portfolio – transfer the base correlation structure from the standard portfolio to the bespoke
- Generally, solve an equation of the form:

$$S(\hat{P}, \hat{u}, \rho) = S(P, u, \rho)$$

S is a "risk statistic"; *P* the portfolio, *u* the detachment point), and *rho* is the base correlation (the unknown is $\hat{u}$).

- Mapping: solve for the detachment point(s) in the bespoke portfolio which matches the equation:
  - Note: the base correlation for the standard portfolio is used on both sides

EL Mapping (Base Correlations)



Understanding Bespoke Portfolios and Prices

- Differences in prices from bespoke portfolio and quoted (index) prices may arise from differences in
  - Credit quality (individual names spreads/PDs)
  - Concentration risk (sector/geographical, and name concentration)
    - Sector concentration (indices are well diversified)
    - Correlation (codependence)
    - Granularity
  - Reference to multiple indices (and different risk premia)
  - Liquidity
- e.g. for an equity tranche:
    - Higher quality (lower individual spreads) → lower tranche spread
    - Higher correlation → lower spread
    - Higher granularity (more names) → higher spread
    - Multiple indices → lower average correlation → higher spread

PART 2

Scenario Framework:

Implied Factor Distributions

and Weighted Monte Carlo Methods

Credit Risk and Multi-Factor Models

- Multi-factor models are currently used extensively to asses portfolio credit risk and measure credit economic capital
  - Required to capture sector/geographical concentrations, and for capital allocation (single-factor models are analytically tractable but limited)
- Industry models go back a decade: KMV, CreditMetrics, CreditRisk+, CreidtPortfolioView
  - Mathematical equivalence – conditional independence framework
  - Computationally efficient methods
- Extensive empirical studies on the historical estimation of credit correlation parameters (Basel committee, rating agencies, vendors, financial institutions and academics)
- The origins of the Gaussian copula method to price CDOs trace back to the KMV and CreditMetrics model

Implied Factor Models & Weighted MC

Background

- Weighted MC approach used to price complex options
  - e.g. Avellaneda et al., 2001, Elices and Giménez, 2006
- Similar idea to fitting the implied distribution (or process) of underlying in a (discrete) lattice
- Hul-White "implied copula" (2006) is essentially an application of this concept
  - Homogeneous portfolio – cannot be used directly to price bespokes
  - Similar ideas (also for homogeneous portfolios) in Brigo et al (2006), Torresetti et al (2006)

Implied Factor Models & Weighted MC

Assumptions

- Correlations of names in portfolio: multi-factor model (systematic factors)
- MF model → joint default behaviour under real world measure *P*
- Coefficients of factor model for portfolio are known and fixed
- Difference between real measure *P* and RN-measure *Q* : joint distribution of the systematic factors
  - (Marginal) distribution of default times for each name under the risk-neutral measure based on CDS spreads
  - Conditional distribution of default times, as a function of the factor levels under the RN measure still given by the same formula

Solution

- Sample discrete "paths" (in this case, single values) for the systematic factors and adjust probabilities of paths to match prices

Weighted MC – *GLLM* Framework

- Portfolio model can be a Gaussian copula or, more generally, we can use other "link functions"
  - Logit, NIG, double-t, etc…
- ***Generalized linear mixed models*** (*GLMM*)

$$p_j^Z(t) = h\left(a(t) - \sum_k b_k Z_k\right)$$

  - Example of general multi-factor copula $p_j^Z(t) = G_j\left(\frac{H^{-1}(F_j(t)) - \sum_k \beta_k Z_k}{\sqrt{1-\sum_k \beta_k^2}}\right)$
  - Match, for each name, the "unconditional" default probability term structure

$$p_j(t) = \int p_j^Z(t)\, df(z)$$

  - … and match quoted CDO prices

Weighted MC – *GLLM* Framework

- General formulation:

$$PD_{it}(Z^t) = h\left(a_{it} + \sum_{k=1}^{K} b_{ik}^t Z_k^t\right)$$

- Gaussian copula:

$$a_{it} = \frac{\Phi^{-1}(PD_{it})}{\sqrt{1-\sum_{k=1}^{K}\beta_k^2}} \quad , \quad b_{ik} = \frac{\beta_k}{\sqrt{1-\sum_{k=1}^{K}\beta_k^2}}$$

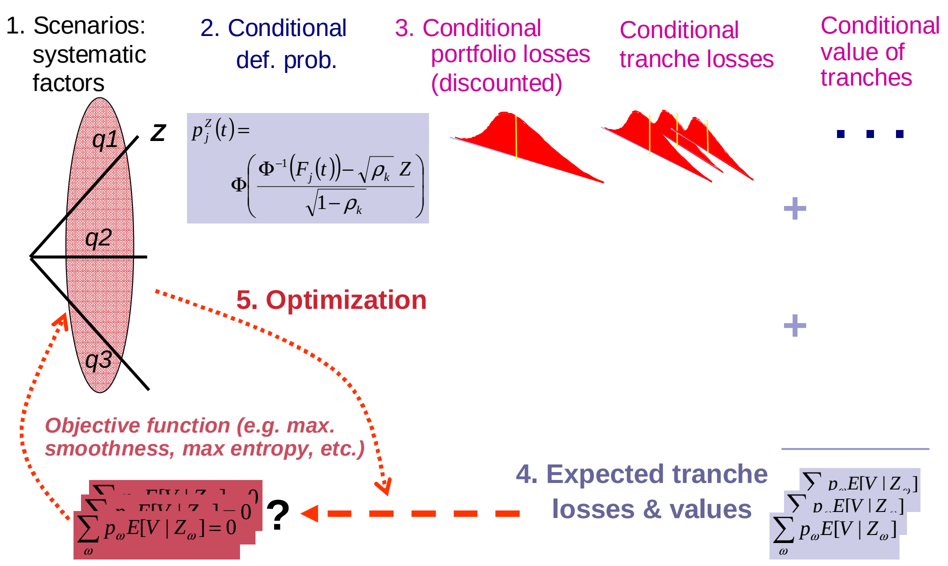
- Poisson mixture (e.g. CreditRisk+)

$$\lambda_i(Z) = E[U_i \mid Z] = c_i \sum_{k=1}^{K} \beta_{ik} Z_k$$

- Logit model:

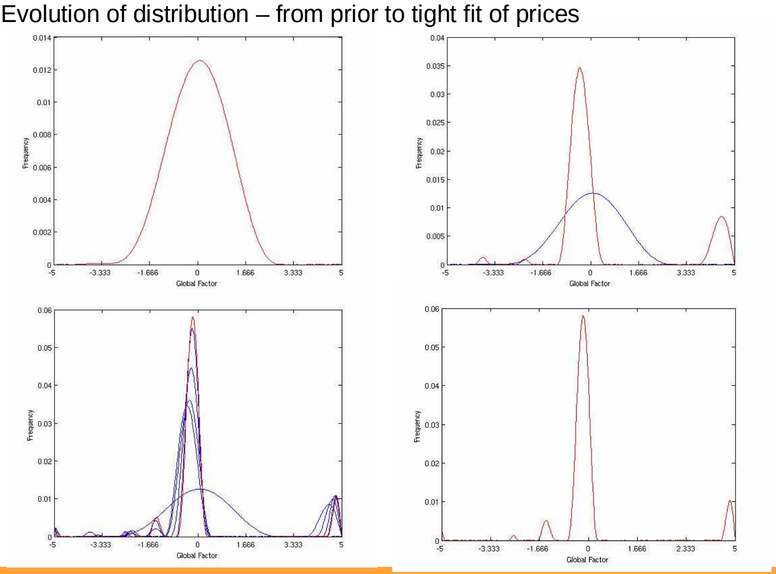
$$h(x) = \frac{1}{(1+\exp(-x))}$$

General Framework: Weighted MC



Global Factor Implied Distribution

Evolution of distribution – from prior to tight fit of prices



Implied Factor Distributions – Intuition

Key objective: tractable distribution of joint default times – match marginal distributions and prices of CDSs and quoted CDO tranches

- In a Gaussian copula – conditioning on the systemic factor

$$p_j^Z(t) = \Phi\left(\frac{\Phi^{-1}(F_j(t)) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)$$

- Base correlations → the correlation *rho* is a function of the detachment point
- Implied copula → model directly conditional PDs through discrete scenarios (on a hazard rate) for homogeneous portfolio
- Implied multi-factor distribution → model directly the ***distribution of the systematic risk factor*** through discrete scenarios → conditional default probabilities through the copula "mapping"
  - Extensible to multi-factor and applied to other portfolios

Weighted MC – Optimization problem

- Objective function → factor distribution "quality":
  - min. distance from prior, max. entropy, max. smoothness
- Match tranche and index prices (can be more than one index at a time)
- Match CDS prices (cumulative default probabilities for all names)

$$\max G(q) \quad \text{subject to :}$$
$$\sum_{m=1}^{M} q_m PV_{Buy}^{n}\left(Z^m\right) = \sum_{m=1}^{M} q_m PV_{Sell}^{n}\left(Z^m\right) \text{ for all } n$$
$$\sum_{m=1}^{M} q_m PD_{i,j}(Z^m) = F_{i,j} \qquad \text{for all } i, j$$
$$\sum_{m=1}^{M} q_m = 1 \ , \quad q_m \geq 0 \quad \text{for all } m$$

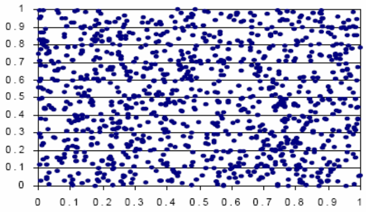
- Trade-off: well-behaved "smooth" solution might be preferred over perfect matching of prices (with some bounds)
  - Instead of perfect "perfect match" – minimize price differences

Weighted MC – Computation

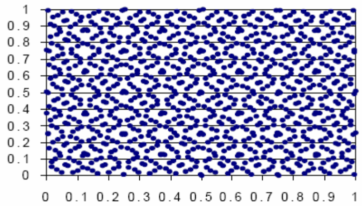
- Computational issues
  - Each path requires computing the conditional portfolio loss distribution – trade-off between number of paths and accuracy of these distributions
  - Number of paths
    - Low dimensions – numerical integration
    - More generally – Quasi-MC methods (low discrepancy sequences)
  - Trade-off: nice behaved "smooth" solution might be preferred over perfect matching of prices (with some bounds of course)

Weighted MC – Scenario Generation

- Quasi Monte Carlo methods
  - Deterministic points generated from a type of mathematical vector sequences: *low discrepancy sequences (LDS)*
- Basic idea:
  - LDSs specifically attempt to cover the space of risk factors “evenly” - avoiding the clustering usually associated with pseudo-random sampling
  - Number of scenarios necessary to achieve a desired level of accuracy in pricing or risk calculations is reduced



(a) Two-dimensional pseudo-random points



(b) Two-dimensional Sobol sequences

Source: Dembo et al. (1999), Mark-to-Future

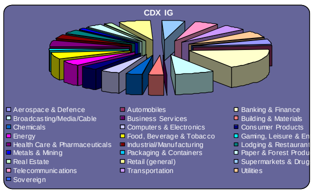
PART 3

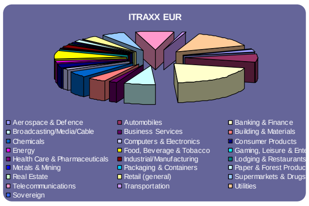
Examples

Example 1: Bespoke Portfolios and Concentration

Industry concentration by Notional

	CDX HY	CDX IG	ITRAXX EUR	ITRAXX CJ	ITRAXX ASIA
Exposure Per Name	10MM	8MM	8MM	20MM	1B
Number of Names	100	125	125	50	50
Industry (Fitch)					
Aerospace & Defence	3.00%	4.00%	2.40%		
Automobiles	9.00%		7.20%	4.00%	4.00%
Banking & Finance	3.00%	17.60%	20.00%	20.00%	24.00%
Broadcasting/Media/Cable	7.00%	8.00%	7.20%		
Business Services	4.00%		1.60%	10.00%	
Building & Materials	4.00%	3.20%	4.00%	6.00%	
Chemicals	6.00%	3.20%	4.80%	2.00%	2.00%
Computers & Electronics	10.00%	4.00%	1.60%	8.00%	6.00%
Consumer Products	5.00%	3.20%	3.20%	4.00%	
Energy	10.00%	4.80%	1.60%		10.00%
Food, Beverage & Tobacco	4.00%	5.60%	7.20%	4.00%	
Gaming, Leisure & Entertainment	3.00%	1.60%	0.00%		2.00%
Health Care & Pharmaceuticals	3.00%	5.60%	0.80%	0.00%	
Industrial/Manufacturing	2.00%	3.20%	2.40%	8.00%	6.00%
Lodging & Restaurants	3.00%	3.20%	0.80%		
Metals & Mining	2.00%	1.60%	0.80%	12.00%	4.00%
Packaging & Containers	2.00%				
Paper & Forest Products	5.00%	3.20%	1.60%		
Real Estate		1.60%	0.00%	4.00%	
Retail (general)	3.00%	6.40%	4.00%	4.00%	8.00%
Supermarkets & Drugstores	1.00%	4.80%	4.80%		
Telecommunications	4.00%	4.80%	8.80%	2.00%	12.00%
Transportation	3.00%	4.80%	0.80%	8.00%	4.00%
Utilities	4.00%	5.60%	14.40%	4.00%	4.00%
Sovereign					14.00%
Herfindahl	5.7%	7.0%	9.5%	9.8%	12.2%
Effective number of sectors	17.5	14.2	10.5	10.2	8.2



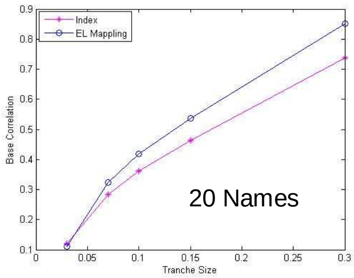


Example1: Bespoke Portfolios and Concentration

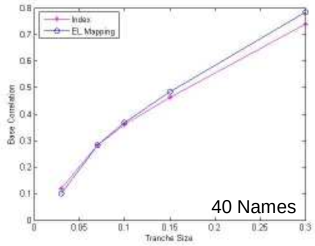
	CDX Index		Bespoke Portfolio (40)		Bespoke Portfolio (20)	
Sector (aggregate)	Weight (Notional)	Weight (EL)	Weight (Notional)	Weight (EL)	Weight (Notional)	Weight (EL)
TECH	20.8%	21.4%	20%	14.5%	**50%**	**72.3%**
SERVICE	9.6%	10.9%	15%	16.7%		
PHARMA	5.6%	3.6%	5%	3.9%		
RETAIL	20.0%	29.0%	17.5%	27.2%		
FINANCE	19.2%	11.8%	10%	6.6%	**50%**	**27.7%**
INDUSTRY	9.6%	9.9%	15%	14.2%		
ENERGY	15.2%	13.5%	17.5%	16.9%		
HI	0.16	0.19	*0.16*	*0.18*	***0.50***	***0.60***
No. Eff. sectors	6.07	5.40	*6.30*	*5.63*	***2.00***	***1.67***

Avg 5yr PD Index = 3.65% 40 Name = 3.92% 20 Name = 3.36%

EL Mapping 20 Names



CDX Point	EL Mapped Point (40)	**EL Mapped Point (20)**
3%	3.45%	**3.17%**
7%	7.01%	**5.89%**
10%	9.75%	**7.98%**
15%	14.01%	**11.55%**
30%	27.77%	**24.65%**

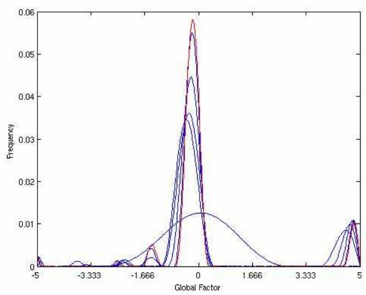


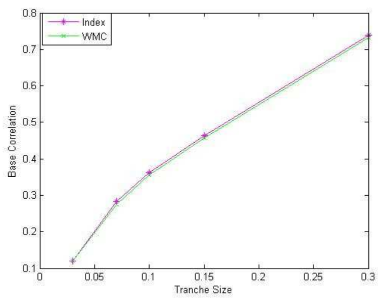
PRICES

Tranche	Index	Bespoke (40)	Bespoke (20)
0 - 3%	31.81%	34.04%	28.26%
3 - 7%	99	139	113
7-10%	21	43	29
10-15%	9.9	18	11
15-30%	5.5	0	0

Example – Weighted MC and CDO Index Prices

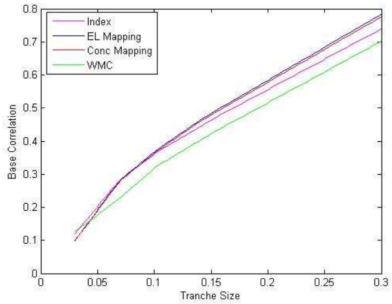
Penalties	Global factor statistics MEAN	STD	SKEW	EX.KURT	Tranche Prices 0-3%	3-7%	7-10%	10-15%	15-30%
None	0.00	0.99	-0.11	-0.27	28.47%	229.06	45.97	9.81	1.12
level 1	0.37	1.90	1.49	1.02	32.31%	118.39	32.85	23.27	9.47
level 2	0.30	1.81	1.68	2.05	32.22%	111.58	34.8	18.67	10.37
level 3	0.22	1.69	1.90	3.33	32.04%	106.54	30.78	16.12	10.77
level 4	0.14	1.54	2.23	5.01	31.89%	101.58	24.46	12.47	7.5
Final	**0.12**	**1.47**	**2.42**	**5.98**	**31.82%**	**99.35**	**21.47**	**10.77**	**5.77**
MARKET					**31.81%**	**99**	**21**	**10.5**	**5.5**





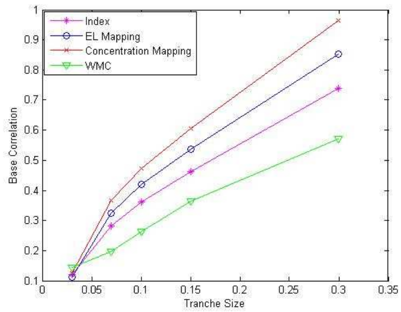
Example – Bespoke (40 Names)

Penalties	Tranche Prices 0-3%	3-7%	7-10%	10-15%	15-30%
None	29.75%	302.4	77.78	19.53	1.83
level 1	31.58%	244.48	39.2	25.16	10.65
level 2	31.93%	232.1	39.48	22.28	10.85
level 3	32.22%	219.38	38.48	19.67	10.99
level 4	32.48%	207.92	35.2	15.41	7.78
Final	**32.57%**	**203.51**	**33.07**	**13.37**	**6.09**
EL Mapping	**34.04%**	**139**	**43**	**18**	**0**
WMC Index	31.82%	99.35	21.47	10.77	5.77
Market Index	31.81%	99	21	10.5	5.5



Example – Bespoke (20 Names)

	0-3%	3-7%	7-10%	10-15%	15-30%
MARKET	***31.81***	***99***	***21***	***10.5***	***5.5***
WMC INDEX	*31.81*	*99.53*	*19.96*	*10.86*	*5.48*
BESPOKE (40 name) WMC	*32.57*	*203.51*	*33.07*	*13.37*	*6.09*
BESPOKE (20 Name) WMC	**20.98**	**266**	**81**	**30**	**8**
BESPOKE EL Mapping	28.26	113	29	11	0



Example 2: Bespoke CDO on Two Indices

Index Prices (5Y)

CDX Tranche	Spread	iTraxx Tranche	Spread
0-3%	0.27617	0-3%	0.11615
3-7%	112.25	3-6%	59.44
7-10%	21.16	6-9%	14.88
10-15%	9.96	9-12%	6.56
15-30%	4.26	12-22%	2.63
Index	38.52	Index	24.33

Implied Default Probabilities

Index	Avg. Hazard Rate	Avg. annual Def. Prob.
CDX	0.00632	0.00629
iTraxx	0.00448	0.00447

Sector Concentrations

Sector	CDX	iTraxx
Comm. and Tech.	14.4%	16%
Financial	18.4%	20%
Materials	8.8%	10.4%
Consumer Stable	12.8%	14.4%
Utilities	6.4%	12%
Energy	4.8%	2.4%
Industrial	11.2%	5.6%
Consumer Cyclical	21.6%	17.6%
Government	1.6%	1.6%
Eff. Num. Sectors	6.92	6.83

(March, 2007)

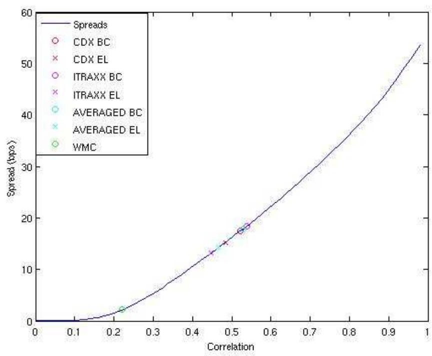
Bespoke CDO – Super Senior (CDX + iTraxx)

Characteristics	Values
Attachment Point	15%
Detachment Point	60%
Underlying Portfolio	100 names, 51 NA (28 CDX, 23 bespoke), 49 Euro (= non NA, 28 iTraxx, 21 bespoke).
Bespoke Average Hazard Rate	0.0115
Bespoke Average 1 Yr. Implied Default Probability	0.0114

Sector	Bespoke	CDX	iTraxx
Comm. and Tech.	17%	14.4%	16%
Financial	14%	18.4%	20%
Materials	19%	8.8%	10.4%
Consumer Stable	11%	12.8%	14.4%
Utilities	2%	6.4%	12%
Energy	1%	4.8%	2.4%
Industrial	3%	11.2%	5.6%
Consumer Cyclical	32%	21.6%	17.6%
Government	1%	1.6%	1.6%
Eff. Num. Sectors	4.985	6.92	6.83

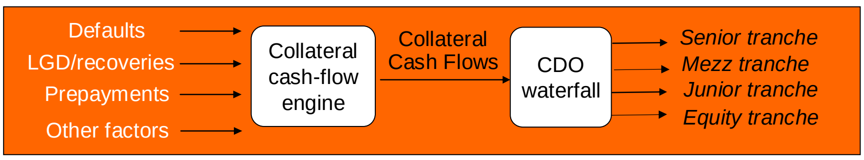
Bespoke CDO Pricing

Method	Bespoke Spread	CDX comp Spread	iTraxx comp Spread
Weighted Monte-Carlo	**2.19 (ρ=0.22)**	**3.78 (ρ=0.41)**	**2.05 (ρ=0.43)**
CDX Base Correlation	17.4 (ρ=0.52)	8.46 (ρ=0.52)	4.56 (ρ=0.52)
CDX EL Mapping	15.25 (ρ=0.49)	6.76 (ρ=0.49)	3.38 (ρ=0.49)
iTraxx Base Correlation	18.38 (ρ=0.54)	9.24 (ρ=0.54)	5.16 (ρ=0.54)
iTraxx EL Mapping	13.16 (ρ=0.45)	5.19 (ρ=0.45)	2.46 (ρ=0.45)
Avg. Base Correlation	17.88 (ρ=0.53)	8.84 (ρ=0.53)	4.85 (ρ=0.53)
Averaged EL Mapping	14.21 (ρ=0.47)	5.96 (ρ=0.47)	2.89 (ρ=0.47)



Example 3: ABS CDO and ABX

- Pricing ABS CDOs is more complex
  - Each underlying bond may contain thousands of small loans (in essence, CDO^2)
  - Cash-flow generation
    - Cashflow of underlying loans in pool
    - Complex CDO waterfall (IR and principal cash-flows, OC, substitution rules, fees, active management, etc.)
- In addition to default (and LGDs), prepayment generally plays an important role (and applies different to different tranches
- Simplistic valuation as bond with deterministic scheduled amortization
  - Computed under a single scenario (default, LGD, prepayment, interest rates, spreads)
  - Discount spread from comparable trades (matrix pricing)



Risky Bond Models for CDOs

- ***Single-scenario*** modelling
  - Deterministic cash-flow approach
- No direct modelling of correlations, optionality and non-linearities in structure
- Detailed cash-flow modelling of collateral pool and of CDO waterfall
  - Pool level assumptions and loan-level assumptions and clustering
- Comparative pricing via matrix approach
  - Scenario assumptions, discount spreads (premiums)
- Efficient stress-testing framework

$$PV = \sum_{j=1}^{T} CF_j(Y) \cdot e^{-(r_j + s_j(X)) \cdot t}$$

$$Y = (PP, Def, LGD, \ldots) \qquad X = (\text{rating, sector, vintage, ...})$$

Stochastic Models for Cash CDOs

- Standard valuation technique for derivatives (option-theoretic approach)
  - Multi-scenario approach (via structured scenarios, MC, analytics)
  - Captures explicitly key risks: credit (default, LGD, spread), prepayment, market risk
  - Portfolio risk – correlation
  - "Arbitrage-free" (hopefully) approach (although can add liquidity, etc.)
- Consistent valuation of various asset classes (synthetics, cash; ABS, CLO, CDO, CDO^2)
- Sensitivities to various risks and hedge ratios
- Computationally intensive
- Various level of modelling
  - Static (e.g. copula) vs. dynamic
  - Top-down vs. bottom-up approaches
  - Risks: credit, prepayment, market, liquidity

Example – ABX

- ABX –referencing 20 Asset Backed CDS (ABCDS)
  - Home Equity / Sub-prime Bonds
  - Five indices: AAA / AA / A / BBB / BBB-
  - Trading Began January 2006
- Standard prices and quotes
- Modelling issues
  - Default risk as well as prepayment risk (competing risks)
  - Cash-flow generation
    - Underlying loans
    - Bond (and CDS) waterfall

ABX	MktPrice
ABX-XHAAA72_INDEX	91.81
ABX-XHAA72_INDEX	71.06
ABX-XHA72_INDEX	44.31
ABX-XHBBB72_INDEX	26
ABX-XHBBBM72_INDEX	23

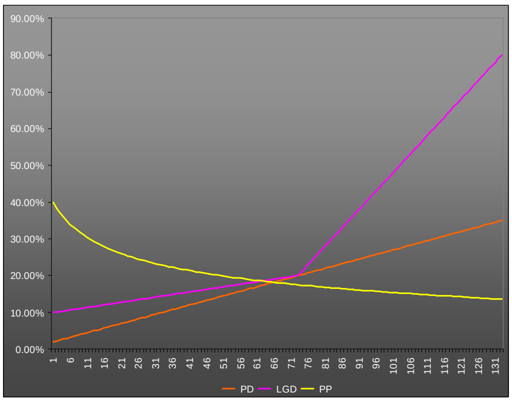
Example – ABX Valuation Model

Simple valuation model (illustration purposes)

$$p_j^Z(t) = h\left(a(t) - \sum_k b_k Z_k\right)$$

- Single systematic factor – drives
  - Default rates
  - Prepayment rates
  - Recovery rates

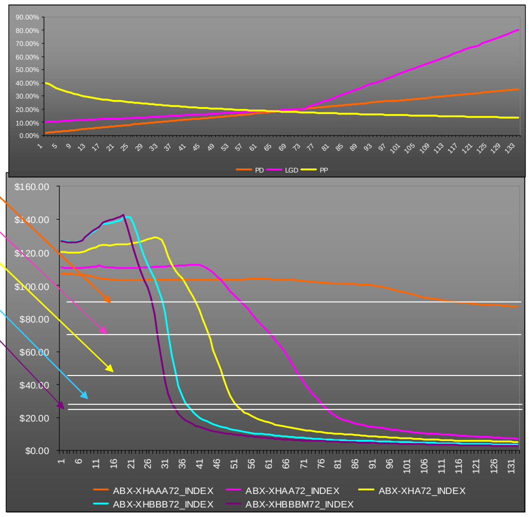
  (underlying loans in the pools)
- Large homogeneous portfolio assumption
- Discretization
  - 133 systematic factor scenarios



Example – ABX Valuation under Scenarios

- ABX bonds discounted Cashflows (values) under scenarios

ABX	MktPrice
ABX-XHAAA72_INDEX	91.81
ABX-XHAA72_INDEX	71.06
ABX-XHA72_INDEX	44.31
ABX-XHBBB72_INDEX	26
ABX-XHBBBM72_INDEX	23



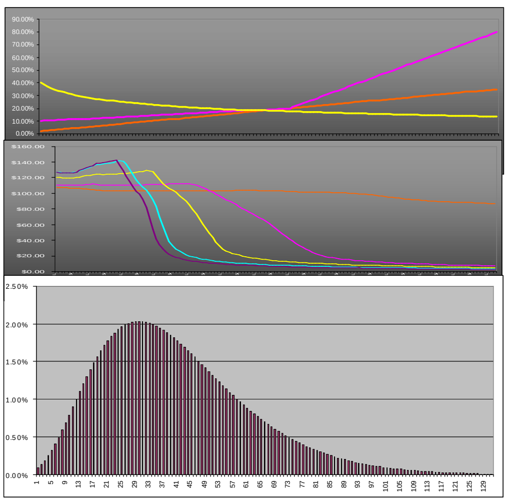
Example – ABX Valuation and Calibration

- Weighted MC → implied factor distribution (implied scenario weights)

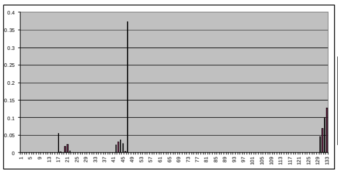
Example:

- Vasicek PD distribution (Gaussian copula GLLM)
  - Implied avg. PD = 12%
  - Implied correlation = 7%

	Market Price	Estimated Prices
ABX-XHAAA72	91.81	103.06
ABX-XHAA72	71.06	94.40
ABX-XHA72	44.31	78.89
ABX-XHBBB72	26	57.19
ABX-XHBBBM72	23	48.47

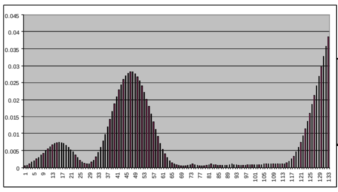


Example – ABX Valuation and Calibration



	Market Price	Model Prices	
ABX-XHAAA72_INDEX	91.81	97.66	5.99%
ABX-XHAA72_INDEX	71.06	69.91	-1.65%
ABX-XHA72_INDEX	44.31	44.69	0.86%
ABX-XHBBB72_INDEX	26	25.84	-0.63%
ABX-XHBBBM72_INDEX	23	23.02	0.08%

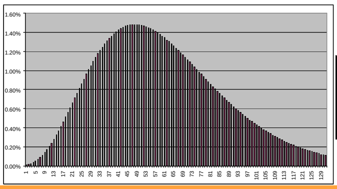
Best fitted prices



	Market Price	Model Price
ABX-XHAAA72_INDEX	91.81	98.46
ABX-XHAA72_INDEX	71.06	69.59
ABX-XHA72_INDEX	44.31	44.94
ABX-XHBBB72_INDEX	26	26.04
ABX-XHBBBM72_INDEX	23	22.70

7.2%
-2.1%
1.4%
0.2%
-1.3%
7.8%

Smoothed distribution (non-parametric)



	Market Price	Model Price
ABX-XHAAA72	91.81	101.46
ABX-XHAA72	71.06	69.07
ABX-XHA72	44.31	45.35
ABX-XHBBB72	26	27.21
ABX-XHBBBM72	23	21.70

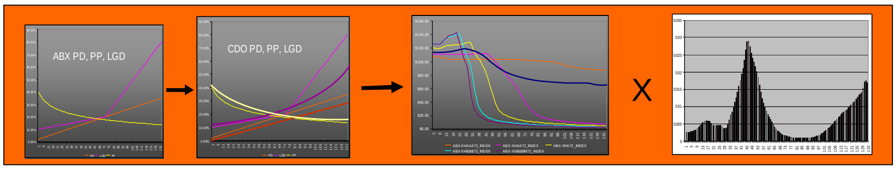
10.5%
-2.8%
2.3%
4.7%
-5.7%
13.3%

Optimal parametric Distribution PD=16.5 R=7%

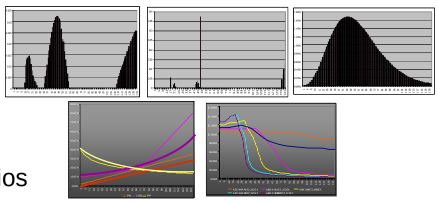
Example – Valuing ABS CDO

Steps

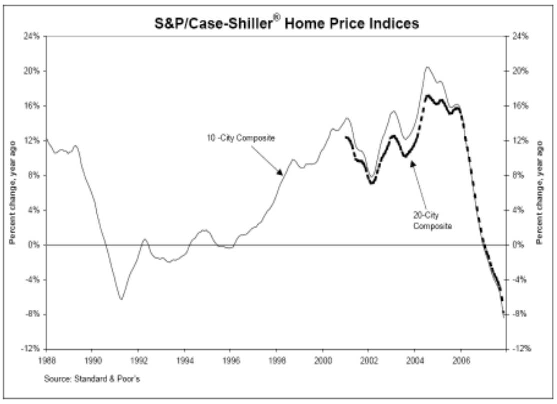
1. Factor scenarios → PD, PP, LGD scenarios for new CDO (factor models)
2. CDO PV scenarios from cash-flow and waterfall engines
3. Valuation using pre-calibrated model



4. Sensitivities and risk measures
5. Model risk assessment
   - "Plausible" factor distributions
   - PD, PP, LGD model assumptions
   - Stress testing and extreme scenarios



Remarks – Systematic Risk and Cash CDOs



- Strong conditional systematic factor → falling home prices
  - Default rates of subprime mortgages
  - Correlation of default of ABS tranches
- Home prices in the US could not continue to increase indefinitely - regime switch
- Default rates and ABS tranche's correlations based on benign period (1996-2006 – prices continually rising) not applicable in a falling price environment

Summary

A valuation/risk framework for structured credit and CDOs requires

- General instrument coverage
  - Index based and bespoke portfolios – CDX/iTraxx (corporate); multiple indices; ABX (ABS securities), and loans
  - Cash-flow structures: synthetic, cash, hybrid; complex structures (e.g. CDO^2)
- Consistent valuation of similar baskets – calibration to observed prices
  - Characterize and model explicitly the effect of portfolio concentrations
  - Arbitrage-free prices (or "comparable risk" valuation)
  - Flexible calibration – fit to prices and makes best use of market, historical and portfolio information effectively
- Practical
  - Transparent, easy to understand and relate to practitioners
  - Computable
  - Lead to natural parameterization and model risk assessment
  - Risk measures

Summary

- Systematic approach – robust and practical CDO valuation framework
  - CDO analytics and computational techniques
  - Multi-factor credit models
  - Weighted Monte Carlo techniques (used in options pricing)
- Value consistently bespoke tranches, CDOs of bespoke portfolios, products on multiple indices, structured finance CDOs, CDO-squared,
  - Arbitrage-free prices
  - Flexible calibration – fits prices and makes effective use of market, historical and portfolio information
  - Characterize and model explicitly the effect concentration/diversification
  - Transparent, easy to understand and relate to market practices
  - Integration of other risks (prepayment) and complex structures (ABS, cashCDO)
- Practical advantages of working through factor models, rather than directly on a common hazard rate (or a set of them)

Presenter's Bio

Dr. Dan Rosen is the co-founder and President of ***R^2 Financial Technologies*** and acts as an advisor to institutions in Europe, North America, and Latin America on derivatives valuation, risk management, economic and regulatory capital. He is a research fellow at the ***Fields Institute*** for Research in Mathematical Sciences and an adjunct professor at the ***University of Toronto****'s* Masters program in Mathematical Finance.

Up to July 2005, Dr. Rosen had a successful ten-year career at *Algorithmics* Inc., where he held senior management roles in strategy and business development, research and financial engineering, and product marketing. In these roles, he was responsible for setting the strategic direction of Algorithmics' solutions, new initiatives and strategic alliances as well as heading up the design, positioning of credit risk and capital management solutions, market risk management tools, operational risk, and advanced simulation and optimization techniques, as well as their application to several industrial settings.

Dr. Rosen lectures extensively around the world on financial engineering, risk and capital management, credit and market risk. He has authored numerous papers on quantitative methods in risk management and applied mathematics, and has coauthored two books and various chapters (including two chapters of PRMIA's Professional Risk Manager Handbook). In addition, he is a member of the Industrial Advisory Boards of the Fields Institute and the Center for Advanced Studies in Finance (CASF) at the University of Waterloo, the Academic Advisory Board of Fitch, the Advisory Board and Credit Risk Steering Committee of the IAFE and the regional director in Toronto of PRMIA. He is also one of the founders of RiskLab, an international network of research centres in Financial Engineering and Risk Management.

He holds an M.A.Sc. and a Ph.D. in Chemical Engineering from the University of Toronto.

Selected Recent Publications

- Rosen D. and Saunders D., 2007, *Valuing CDOs of Bespoke Portfolios with Implied Multi-Factor Models*, Working Paper Fields Institute for Mathematical Research and University of Waterloo
- Rosen D. and Saunders D., 2006a, *Analytical Methods for Hedging Systematic Credit Risk with Linear Factor Portfolios*, Working Paper, Fields Institute for Mathematical Research and University of Waterloo
- Rosen D. and Saunders D., 2006b, *Measuring Capital Contributions of Systemic Factors in Credit Portfolios*, Working Paper Fields Institute for Mathematical Research and University of Waterloo
- Garcia Cespedes J. C., Keinin A., de Juan Herrero J. A. and Rosen D., 2006, *A Simple Multi-Factor "Factor Adjustment" for Credit Capital Diversification*, Special issue on Risk Concentrations in Credit Portofolios (M. gordy, editor) Journal of Credit Risk, Fall 2006
- Garcia Cespedes J. C., de Juan Herrero J. A., Rosen D., Saunders D., 2007, *Effective modelling of Alpha for Regulatory Counterparty Credit Capital*, Working Paper, Fields Institute
- Mausser H. and Rosen D., 2007, *Economic Credit Capital Allocation and Risk Contributions*, forthcoming in Handbook of Financial Engineering (J. Birge and V. Linetsky Editors)
- De Prisco B., Rosen D., 2005, *Modelling Stochastic Counterparty Credit Exposures for Derivatives Portfolios*, Counterparty Credit Risk (M. Pykhtin, Editor), Risk Books, London
- Aziz A., Rosen D., 2004, *Capital Allocation and RAPM*, in Professional Risk Manager (PRM) Handbook, Chapter III.0, PRMIA Publications
- Rosen D., 2004, *Credit Risk Capital Calculation*, in Professional Risk Manager (PRM) Handbook, Chapter III.B5, PRMIA Publications

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