Stochastic Programming Models for International Asset Allocation Problems

Hercules Vladimirou
Nikolas Topaloglou, Stavros Zenios

HERMES Center on Computational Finance & Economics
School of Economics & Management
University of Cyprus

RiskLab Meeting

Madrid, Dec. 1, 2003

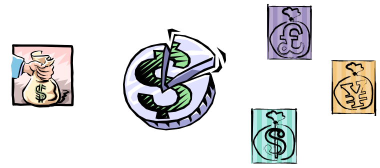
Points of Discussion:

- **Problem Issues**
  - Problem framework & risk factors **(Market & Currency Exchange Risk)**
  - Diversification & Hedging policies
  - Risk management metrics
- **Modeling Approaches**
  - Scenario Generation
  - Optimization Models (Stochastic Programs)
    (jointly determine portfolio composition and hedging levels in each market – selective hedging via forwards and options)
  - Incorporation of Options in Portfolio
- **Empirical Assessment of Models & Investment Strategies**
  - Risk/Return Profiles of Portfolios (static tests)
  - Out-of-sample Performance (Consistency)
  - Backtesting (Ex-post performance)

International Portfolio Management:

The Problem:

Allocation of funds to international assets
Dynamic management of portfolio



The Objectives:

Effective Management of Risk/Return Tradeoffs (parametric programs)
Diversification & Risk Hedging

The Needs:

- Representation of uncertainty capturing market & exchange rate randomness
- Constistent pricing of Options
- Portfolio Optimization Models utilizing suitable risk measures
to control total risk exposure

International Diversification

- It pays to diversify internationally
- Positive empirical evidence holds for portfolios of equities and bonds
- Intl. diversification entails additional risks (currency exchange fluctuations)
  - Eun & Resnick, *J. of Finance*, 1988.
- Higher correlations of international investments in bear markets

Effects of international diversification?

They depend on the volatility and correlation structures of the international markets and currency exchange rates.
International diversification entails exposure to currency risk.
Eun, Resnick, *Journal of Finance*, 1988.

Observations:

- General ***increase*** in local return correlations
- Volatility is ***contagious*** across markets
- Higher correlation in bear markets

} Market Synchronization & Interdependencies

- Majority of pension funds invested abroad, are managed as overlays portfolios (*Pension and Investment Age*, 1993)
- Currency risk (partly) hedged with forward currency exchanges
- Derivative securities - alternative risk management means

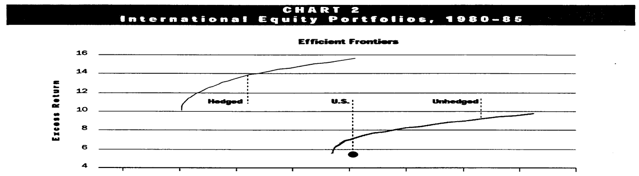
  (to hedge either market risk, or exchange risk, or both)

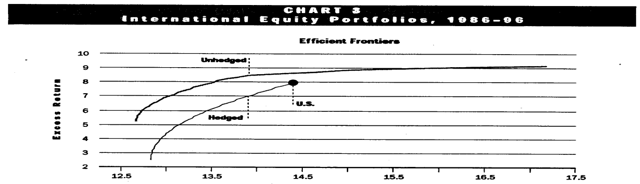
Holistic risk management tools employed.

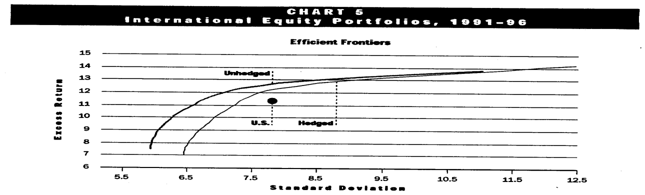
To Hedge or Not to Hedge Currency Risk?

- Perold and Shulman, *Financial Analysts Journal*, 1988:
  Yes! **Free *lunch*** in currency hedging!
- Kaplanis and Schaefer, *J. of Economics and Business*, 1991:
  Some times Yes and some times No, else we don't know!
- P. Jorion, *J. of Portfolio Management*, 1989:
  Some times Yes, some times No, else we need to determine a hedge ratio!
- F. Black, *J. of Finance*, 1990:
  Universal hedge ratio for all investors and all foreign holdings.
- Filatov and Rappaport, *Financial Analysts Journal*, 1992:
  Some times Yes, some times No, else we have a hedge ratio!
- Abken and Shrikhande, *Federal Reserve Bank of Atlanta Economic Review*, 1997:
  Course of action influenced by various factors.
- Beltratti, Laurent and Zenios, "*Scenario Modeling of Selective Hedging Strategies*", *JEDC*, 2003:
  Selective Hedging is the Preferred Strategy!
  We also formulate implementable hedging policies.

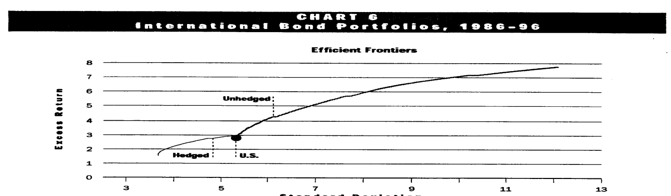
  Single period MAD model   Historical observations used as scenarios

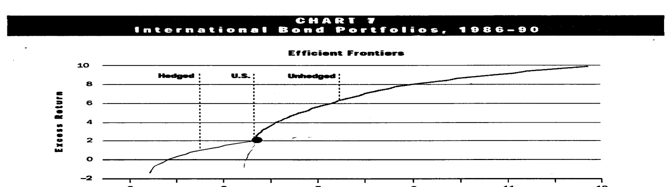


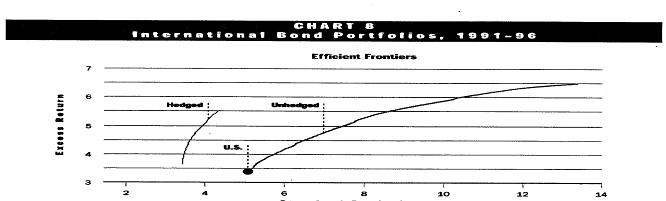




Abken and Shrikhande, *Federal Reserve Bank of Atlanta Economic Review*, 1997.







Abken and Shrikhande, *Federal Reserve Bank of Atlanta Economic Review*, 1997.

Factors found empirically to affect the performance of alternative hedging policies

The literature presents different views as to the optimal course of (currency hedging) action for international portfolio management depending on factors such as:

- Investment opportunity set
- Investor's reference currency denomination
- Representation of uncertainty
- Timeframe of study (calibration data)
- Investor's time horizon and risk taking criteria
- Investment strategy (static vs dynamic)

- Endogenize currency hedging decisions in portfolio construction model

Selective Hedging: Integrative framework

Endogenize hedging decisions in portfolio selection procedure

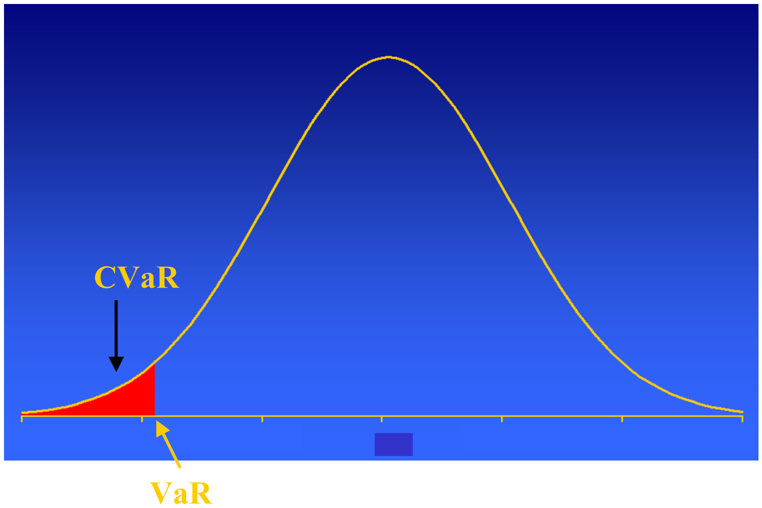
- Scenarios of index domestic returns and exchange rates capturing correlations between them
  (define scenarios of holding period returns)
- Currency hedging via forward exchanges and/or options
  (alternatives for controlling hedging decisions)
- Portfolio optimization models determine portfolio compositions and currency hedging levels

Extensions/Contributions:

- CVaR risk measure (more appropriate for skewed distributions, coherent)
- Scenario generation procedures (& Stability investigation)
- Operationalization of hedging decisions (specification of forward contracts)
- Introduction of options in portfolio optimization models

Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR)

Portfolio Value Distribution at horizon *T*



The Problem:

- International asset-allocation problem
  (single- and two-stage SP models; monthly time steps)

Assets:

- Stock Indices in various countries (USD, GBP, DEM, JPY)
- Government Bond Indices in various countries
  - Short-term bonds (1-3 years)
  - Intermediate-term bonds (3-7 years)
  - Long-term bonds (7-10 years)
- Options:
  Stock Index Options, Quantos, Currency Options

Data Sources:

- Morgan-Stanley MSCI Data (Stock Indices)
- Datastream
  - Salomon Brothers Government Bond Indices
  - Spot & Forward Exchange Rates

Descriptive Statistics of Historical Data	Mean	St.Dev.	Skewness	Kurtosis
USS	1.519%	3.900%	-0.465	4.271
UKS	1.164%	4.166%	-0.233	3.285
GRS	1.213%	5.773%	-0.511	4.503
JPS	-0.133%	6.336%	0.022	3.609
US1	0.537%	0.473%	-0.144	2.801
US7	0.688%	1.646%	-0.047	3.276
UK1	0.723%	0.710%	1.330	7.209
UK7	0.913%	1.932%	0.108	3.482
GR1	0.537%	0.458%	0.655	5.319
GR7	0.670%	1.390%	-0.863	4.482
JP1	0.327%	0.522%	0.492	4.147
JP7	0.608%	1.731%	-0.514	5.149
UStoUK	-0.074%	0.081%	-1.084	6.790
UStoGR	-0.167%	0.088%	-0.398	3.908
UStoJP	0.303%	0.133%	1.123	6.904

Generally, asset returns are not normal; exhibit asymmetries and fat tails.
Motivation for:

- **alternative scenario generation procedures**
- **risk metrics suitable for asymmetric distributions (i.e.,CVaR)**
- **alternative option pricing procedure**

Issues:

• Development of CVaR models for international asset allocation that jointly determine the level of currency hedging and select the appropriate investments

• Pricing of options on assets/currencies consistently with postulated scenarios

• Incorporating options (currency options, stock options) in international portfolios of stocks and bonds

• Hedging currency and market risk jointly using Quantos

• Assessment of alternative trading strategies involving combinations of options (strip, strap, straddle, strangle).

Scenario Generation:

1. Principal Component Analysis (PCA)

- Calibrated using historical market data
- Directed selective sampling from empirical distributions of PCs
- Bayes-Stein estimation corrections
- Difficult extension to multistage scenario trees
- Do not match all statistical characteristics

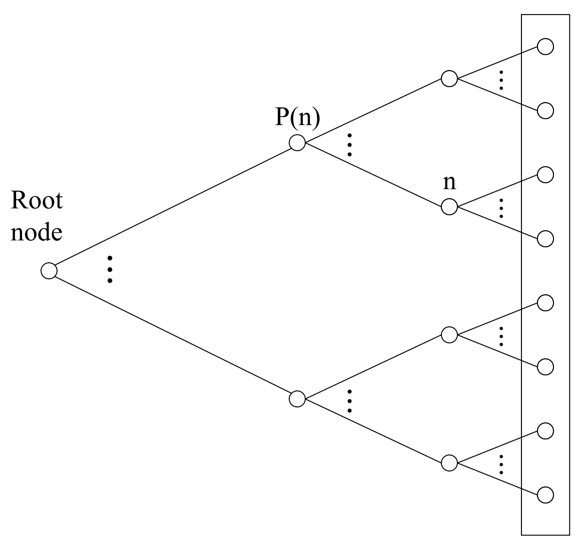
2. Moment-Matching Scenario Generation Methods

(Hoyland, Wallace) – ***Management Science***, 2002
(Hoyland, Wallace, Kaut) – ***Comp. Optim. & Appl.***, 2003

- First four marginal moments & correlations match target values
- Targets estimated using historical data
- Scenario tree constructions

Model calibration: 12 past years (rolling horizon)

Scenario Trees



Generic Multistage SP Formulation for International Portfolio Management Problem

Asset inventory constraints:

$$w_{ik}^{0} = h_{ik} + x_{ik}^{0} - y_{ik}^{0} \qquad \forall k \in C, \forall i \in I_k$$

$$w_{ik}^{n} = w_{ik}^{p(n)} + x_{ik}^{n} - y_{ik}^{n} \qquad \forall k \in C, \forall i \in I_k, \forall n \in N \setminus \{0, L\}$$

Cash balance (base currency):

root node

$$c_k^0 + \sum_{i \in I_k} y_{ik}^0 P_{ik}^0 (1 - \zeta_i) + \sum_{c \in C_f} v_c^{s0} (1 - \varsigma_c) = \sum_{i \in I_k} x_{ik}^0 P_{ik}^0 (1 + \zeta_i) + \sum_{c \in C_f} v_c^{b0} (1 + \varsigma_c) \qquad k = C \setminus C_f$$

remaining nodes but not leaves

$$c_k^n + \sum_{i \in I_k} y_{ik}^n P_{ik}^n (1 - \zeta_i) + \sum_{c \in C_f} v_c^{sn} (1 - \varsigma_c) + v_c^{fp(n)} = \sum_{i \in I_k} x_{ik}^n P_{ik}^n (1 + \zeta_i) + \sum_{c \in C_f} v_c^{bn} (1 + \varsigma_c) \qquad k = C \setminus C_f, n \in N \setminus \{0, L\}$$

Generic Multistage SP Formulation for International Portfolio Management Problem

Cash balance (foreign currencies):

root node

$$c_k^0 + \sum_{i \in I_k} y_{ik}^0 P_{ik}^0 (1 - \zeta_i) + \frac{1}{e_k^0} v_k^{b0} (1 - \varsigma_k) = \sum_{i \in I_k} x_{ik}^0 P_{ik}^0 (1 + \zeta_i) + \frac{1}{e_k^0} v_k^{s0} (1 + \varsigma_k) \qquad k \in C_f$$

remaining nodes but not leaves

$$c_k^n + \sum_{i \in I_k} y_{ik}^n P_{ik}^n (1 - \zeta_i) + \frac{1}{e_k^n} v_k^{bn} (1 - \varsigma_k) = \sum_{i \in I_k} x_{ik}^n P_{ik}^n (1 + \zeta_i) + \frac{1}{e_k^n} v_k^{sn} (1 + \varsigma_k) + \frac{1}{e_k^{p(n)}} v_k^{fp(n)} \qquad k \in C_f, n \in N \setminus \{0, L\}$$

Asset Sale Limits

$$0 \le y_{ik}^0 \le h_{ik} \qquad \forall k \in C, \forall i \in I_k$$

[illegible]

Generic Multistage SP Formulation for International Portfolio Management Problem

Initial Portfolio Value

$$V_0 = \sum_{k \in C} \left\{ c_k^0 + \sum_{i \in I_k} h_{ik} P_{ik}^0 \right\} e_k^0$$

Final Portfolio Value

$$V_n = c_k^n + \sum_{i \in I_k} w_{ik}^{p(n)} P_{ik}^n + \sum_{c \in C_f} \left\{ v_c^{fp(n)} + e_c^n \left(c_c^n + \sum_{i \in I_c} w_{ic}^{p(n)} P_{ic}^n - \frac{v_c^{fp(n)}}{f_c^{p(n)}} \right) \right\} \quad n \in L, k \in C \setminus C_f$$

Portfolio Return

$$R_n = \frac{V_n}{V_0} - 1 \qquad n \in L$$

Parametric bound on Expected Portfolio Return

$$\sum_{n \in L} \pi_n R_n \geq \mu$$

Generic Multistage SP Formulation for International Portfolio Management Problem

CVaR definition

$$y_n^+ \geq z - R_n \qquad n \in L$$

$$y_n^+ \geq 0 \qquad n \in L$$

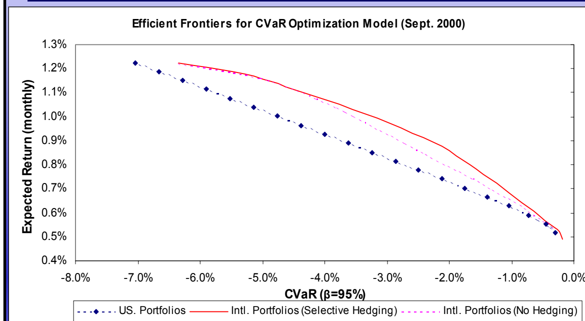
Objective Function:

Maximize $\quad z - \frac{1}{1-\beta} \sum \pi_n y_n^+$

z **: the VaR of portfolio return (at (1-β) percentile)**
the objective value is the respective CVaR

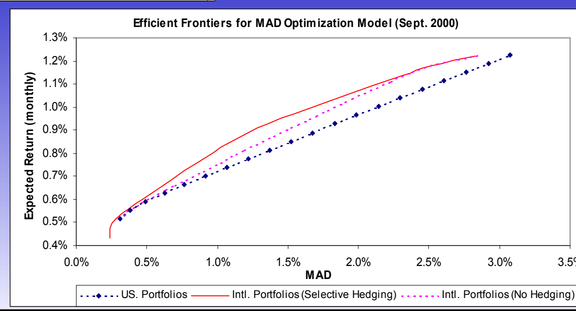
Development of CVaR models:
S. Uryasev and T. Rockafellar (2000-2002),
Journal of Risk, Financial Engineering News, Journal of Banking and Finance, etc.

Potential benefits from international diversification

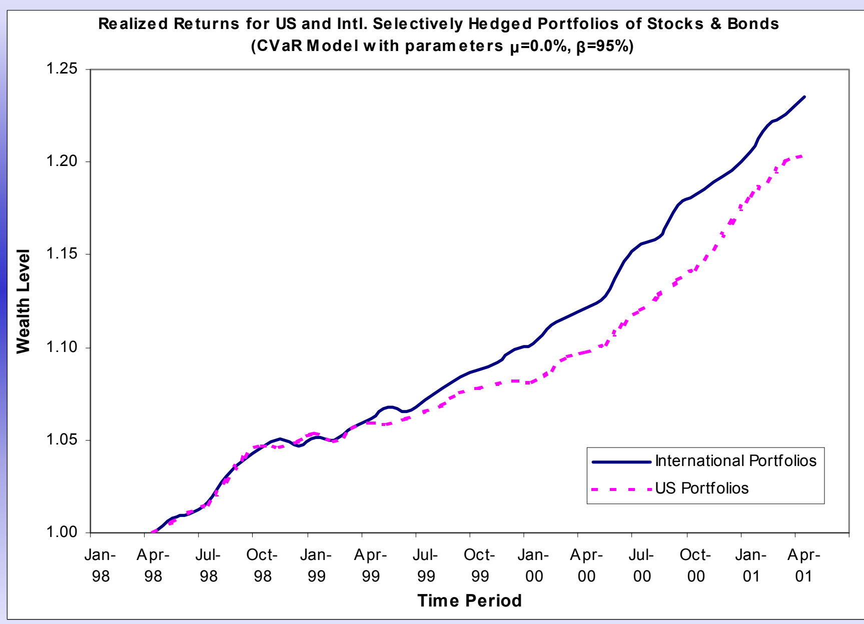


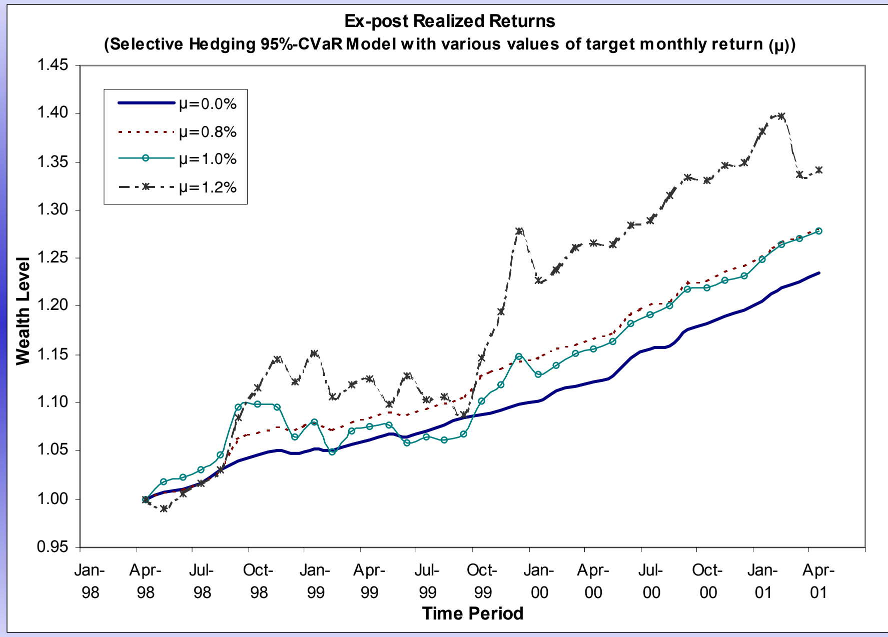
Improvement in risk-return profiles regardless of risk metric (preferable strategy: selective hedging)

Benefits more evident for intermediate risk-tolerance levels.

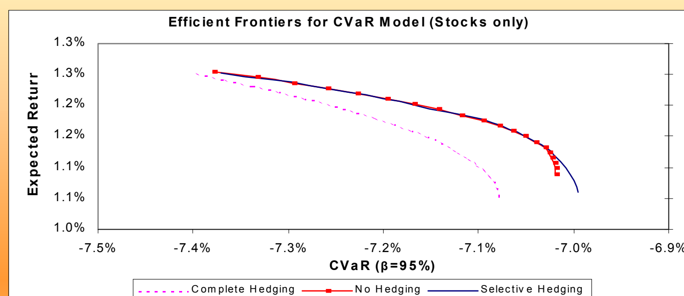


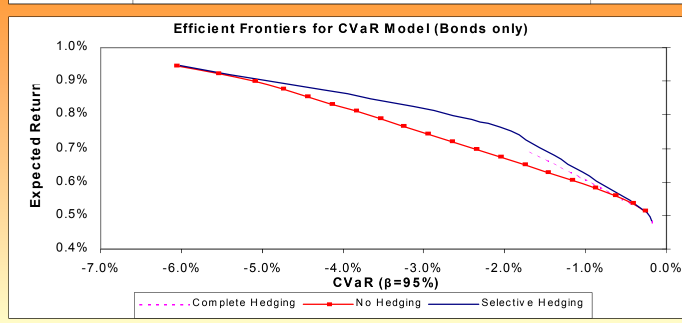
Ex-post benefits from international diversification.



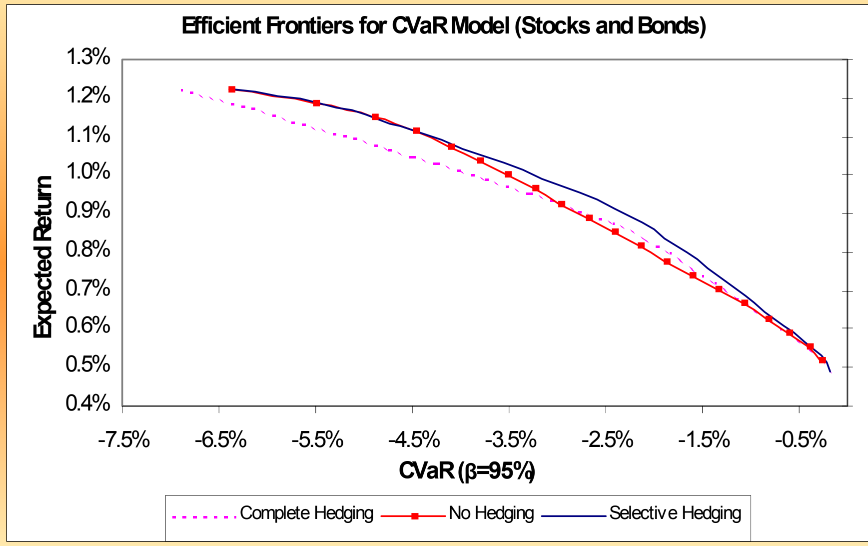


Comparison of Hedging Policies (CVaR Model)





Comparison of Hedging Strategies (CVaR Model)



Selective Hedging is the more effective (flexible) strategy.

Single- or Two-Stage SP Model? (Comparison)

In static tests (Ex-ante):

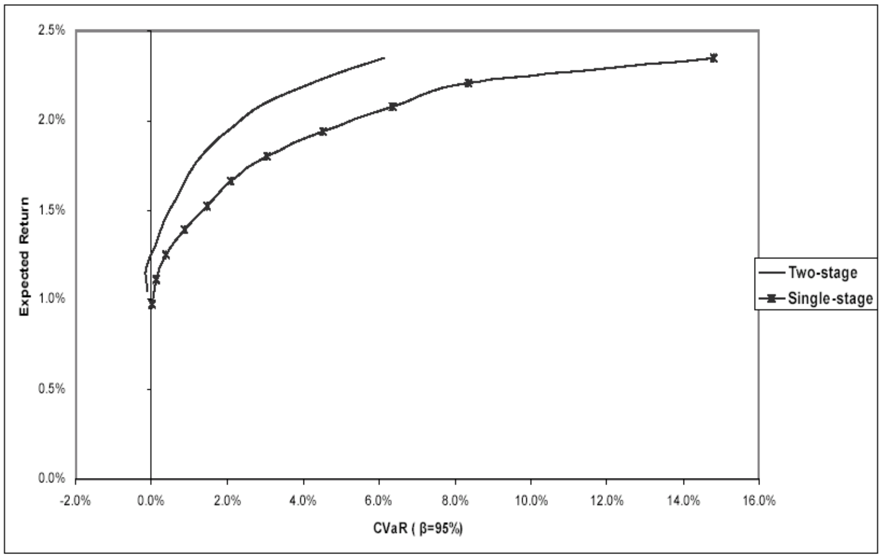
Two-stage SP model is clearly superior to single-stage (Dominant efficient frontiers)

In backtesting experiments (ex-post):

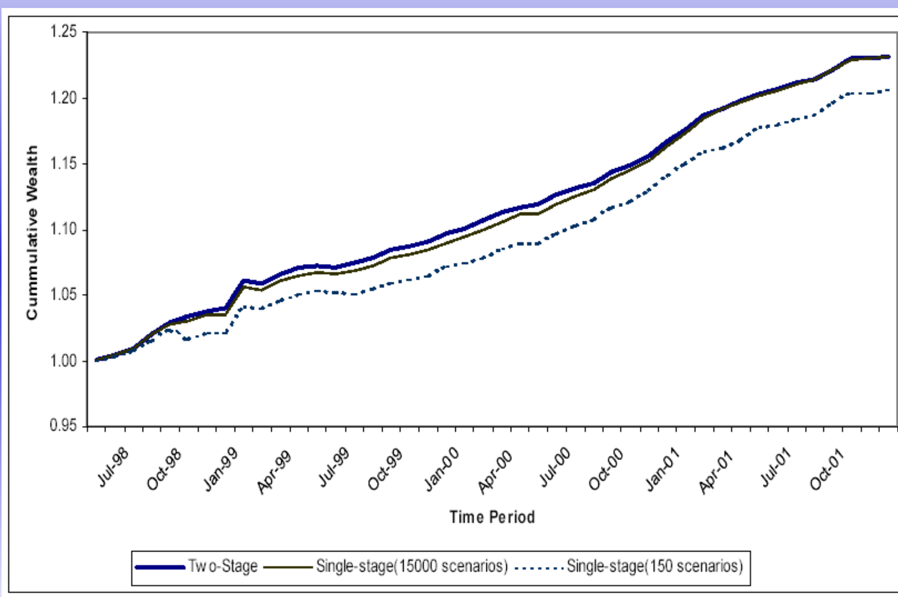
The models exhibit similar performance/behavior
No dominating model can be indisputably identified

1-stage model affords finer representation of short-term uncertainty, while
2-stage model captures effects of subsequent period(s)

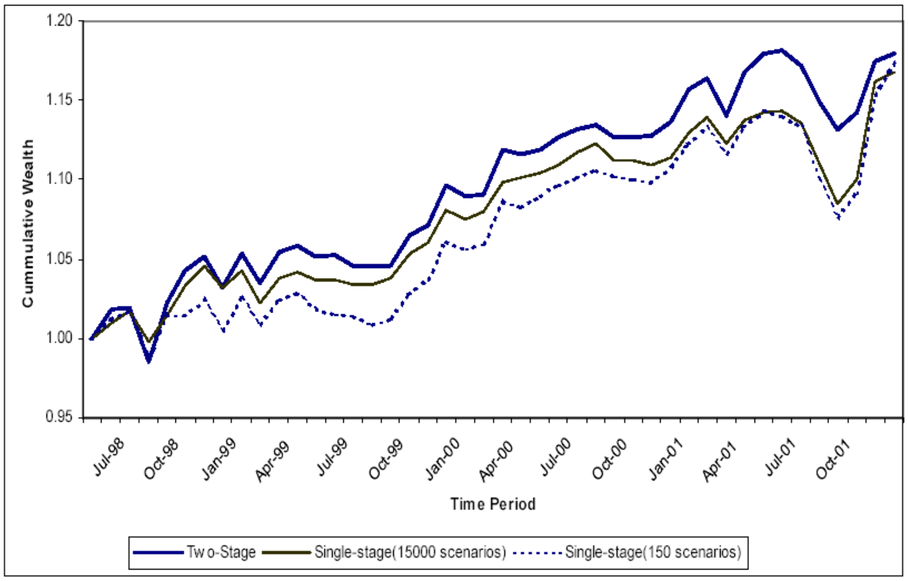
Ex-ante Comparison of Single- & Two- Stage Models



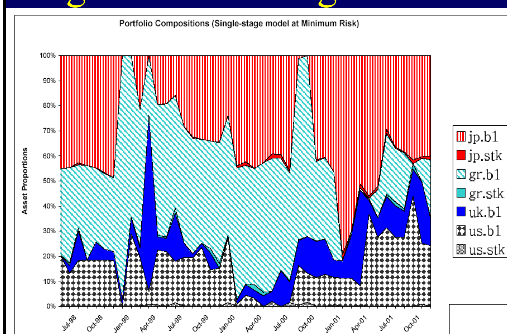
Ex-post Comparison of Single- & Two-Stage SP Models (min risk case)

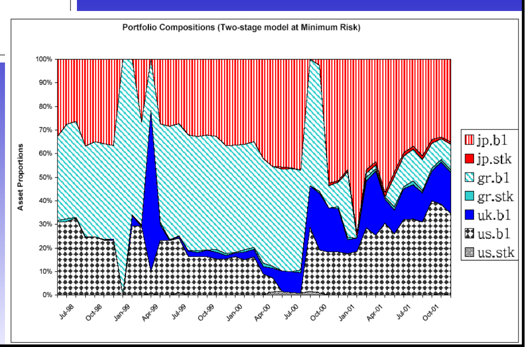


Ex-post Comparison of Single- & Two-Stage SP Models (more aggressive case)

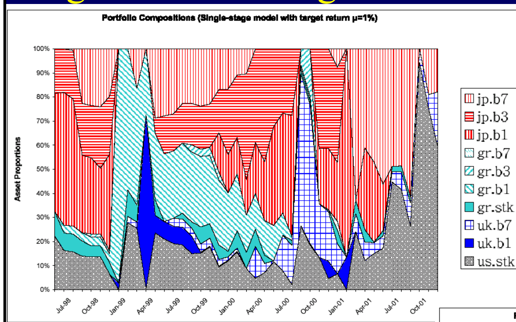


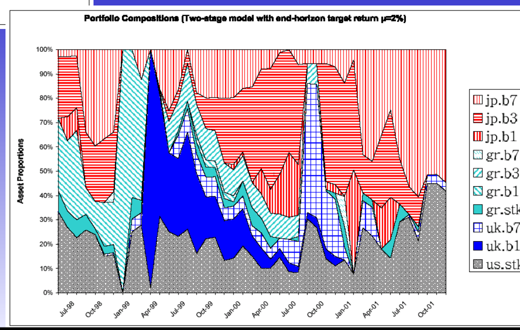
Portfolio Compositions of Single- & Two-Stage SP Models (Minimum Risk Case)





Portfolio Compositions of Single- & Two-Stage SP Models (More Aggressive Case)





Incorporating Options in Portfolios:

- **Introduction of options in portfolio**

(European options with maturity matching rebalancing frequency)

  Options on stock indices

  Quantos on foreign stock indices

  Currency options

- **Options priced consistently with postulated scenario sets and satisfying arbitrage-free conditions**

- **Investigation of alternative risk management (hedging) strategies**

Methodologies for pricing Options

- Expansion methods: Start with a basic distribution and add correction terms (Corrado and Su (J. Financial Research, 1996), Jarrow and Radd (JFE, 1982))

  (Currency Options)
- Derivation of the risk-neutral measure using utility functions: Risk-Neutral Prob = Actual Prob × Pricing Kernel

  (Rosenberg and Engle, WP-2003, Bakshi et al, RFS 2003, Jackwerth, J. Derivatives, 1999, JF 2000)

  (Stock Options)

Incorporating Options in International Portfolios

Investments in different classes of options:

A. **Quantos**: **Fixed exchange rate foreign equity options. Relevant for jointly managing foreign market risk and exchange rate risk.**

Payoff of a call Quanto in reference currency:

C = Max (S X – K , 0)

B. **Simple Options**: **Relevant for managing foreign market risk, but unconcerned about exchange rate risk.**

Payoff of a call option in foreign currency:

C = Max (S – K , 0)

C. **Currency Options**: **Rights for currency exchanges at prespecified rates at option's expiration.**

Incorporating Options in International Portfolios

Pricing of Options

(Consistently with postulated scenario sets)

- Determine a new (risk neutral) probability measure on postulated scenario set based on Radon-Nikodym principle; satisfy martingale property.
- The price of an option is the expected value (under the risk neutral probability measure) of discounted (with riskless rate) payoffs at maturity.
- Currency options priced using procedure of Corrado & Su.
- No-arbitrage conditions verified.

Pricing Currency Options

$$x_{t+1} = \log E_{t+1} - \log E_t \Rightarrow E_{t+1} = E_t e^{x_{t+1}}$$

Then for the option on E we have

$$c = e^{-rt} \mathrm{E}_Q (E_{t+1} - K)^+ = e^{-rt} \int_{\log(K/E)}^{\infty} (E_t e^x - K) f(x) dx$$

If x is not normal then we use Gram-Charlier expansion. It generates an approximate density function for a s.r.v. with nonzero skewness and excess kurtosis:

$$\omega = \frac{x_{t+1} - \mu_t}{\sigma_t}$$

The Gram-Charlier expansion considers approximation density:

$$f(\omega) = n(\omega) - \gamma_1 \frac{1}{3!} D^3 n(\omega) + \gamma_2 \frac{1}{4!} D^4 n(\omega)$$

Pricing Currency Options

$$c = E_0 e^{-r_f T} N(d) - K e^{-r_d T} N(d-\sigma_T) +$$

$$E_0 e^{-r_f T} n(d)\sigma_T [\frac{\gamma_1}{3!}(2\sigma_T - d) - \frac{\gamma_2}{4!}(1 - d^2 - 3d\sigma_T - 3\sigma_T{}^2)]$$

where

$$\gamma_1 = \frac{k_3}{(k_2)^{3/2}} \ldots, \gamma_2 = \frac{\kappa_4}{(\kappa_2)^2} \ldots, n(d) = (2\pi)^{-1/2} e^{-d^2/2}$$

$$d = \frac{\log(E_0 / K) - (r_f - r_d)T + \sigma_T{}^2 / 2}{\sigma_T}$$

Accounts for skewness and kurtosis, deviations from normality of exchange rates

Pricing Quantos and Simple options

Radon-Nikodym theorem: Statement about two equivalent probability measures:

- The actual measure P
- The Risk-Neutral measure Q

on some measurable space Ω.

Radon-Nikodym theorem assertion:

Q(dω)=ξ(ω)P(dω)

Where ξ(ω) is a measurable function with respect to the underlying sigma field.

Pricing Quantos and Simple Options

Hypothesis of power utility function

By normalizing and changing variables

(Bakshi et al, *Review of Financial Studies,* 2003)

The risk-neutral probabilities for each scenario are

$$q^n(R^n) = \frac{e^{-\gamma R^n} p^n(R^n)}{\sum_n e^{-\gamma R^n} p^n(R^n)}$$

Where γ is the relative risk aversion

Pricing Quantos and Simple Options

- Risk-neutral index density obtained by exponentially tilting the physical density.

- To avoid arbitrage, the index must be a martingale
- Mean of risk-neutral density must satisfy martingale condition:

$$S_0 e^{(r-\delta)\mathrm{T}} = \sum_n q^n S^n$$

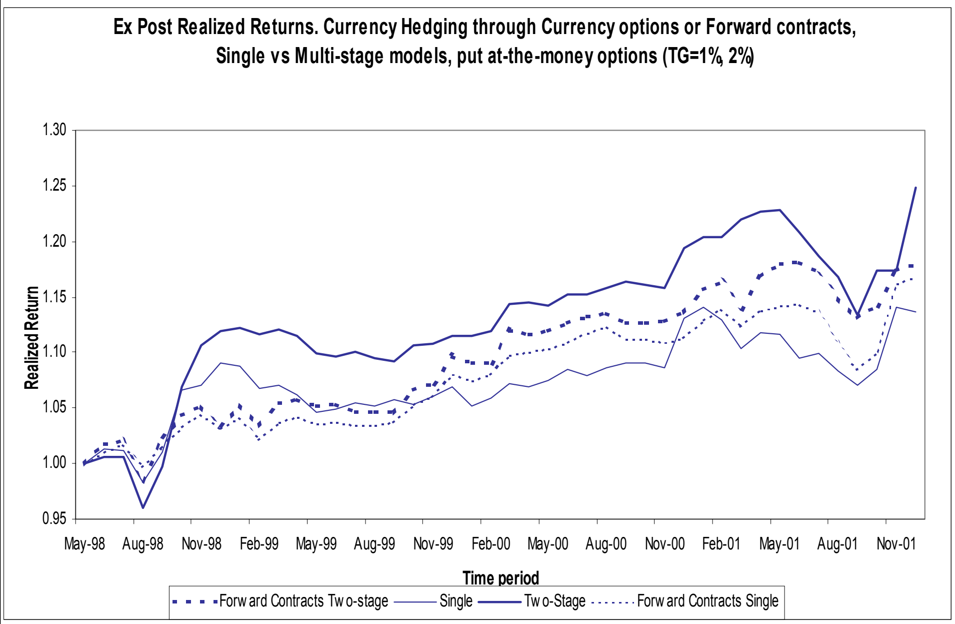
Incorporating Derivatives in International Portfolios

Risk Neutral Valuation:

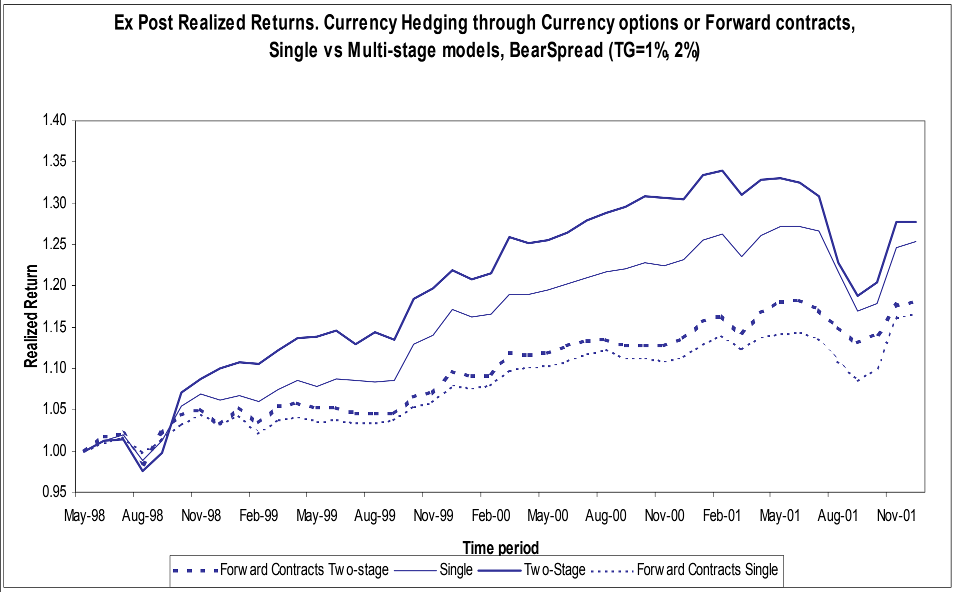
- **The price of the option on asset S is the expected payoff of the option under all scenarios, in Risk-Neutral Measure, discounted at the risk-free rate. Thus, for a Quanto Call:**

$$c = e^{-rdt} \sum_{n=1}^{N} \bar{p}_n \max(XS_n - K, 0)$$

Currency options for hedging purposes

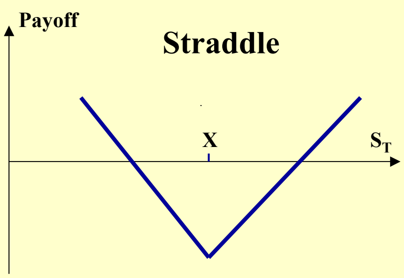


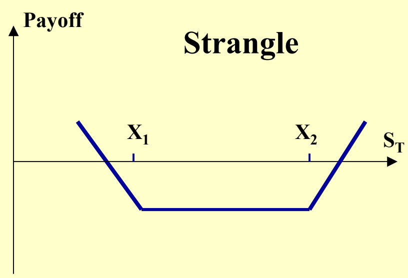
Currency options for hedging purposes



Incorporating Derivatives in International Portfolios

Trading Strategies involving Options

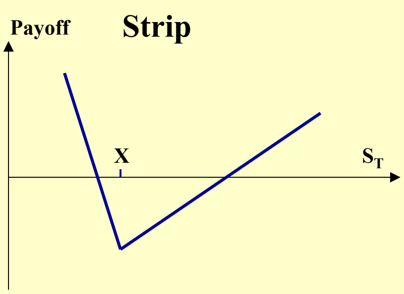


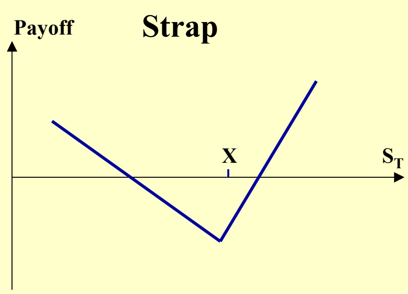


1 Long Call and 1 Long Put with the same exercise price X and the same maturity

1 Long Call (exercise price X_2)
1 Long Put (exercise price X_1)
with the same maturity

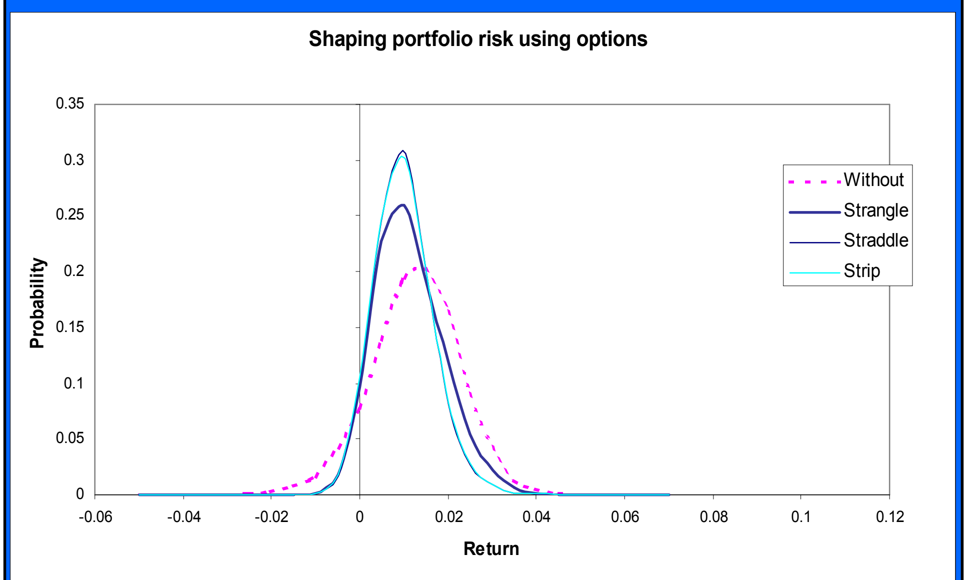
Incorporating Derivatives in International Portfolios

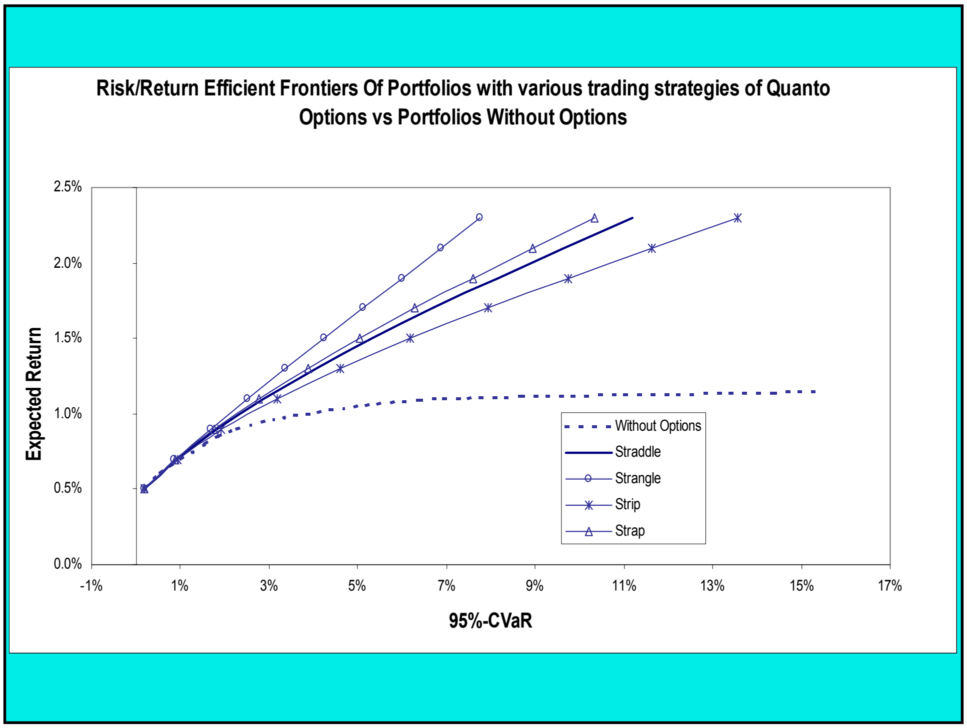


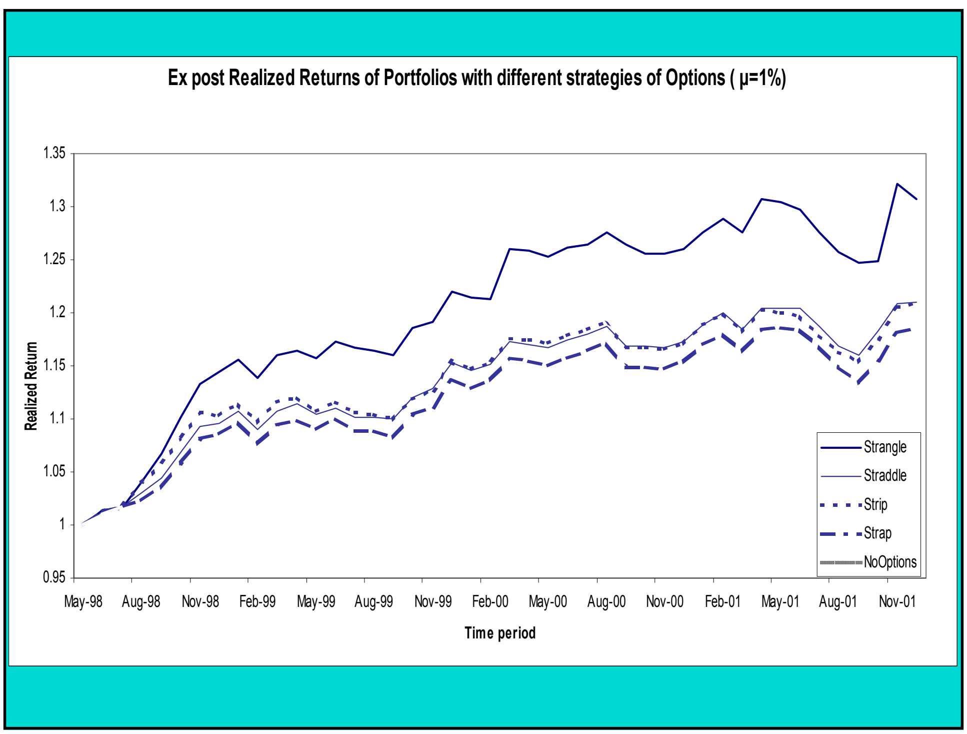


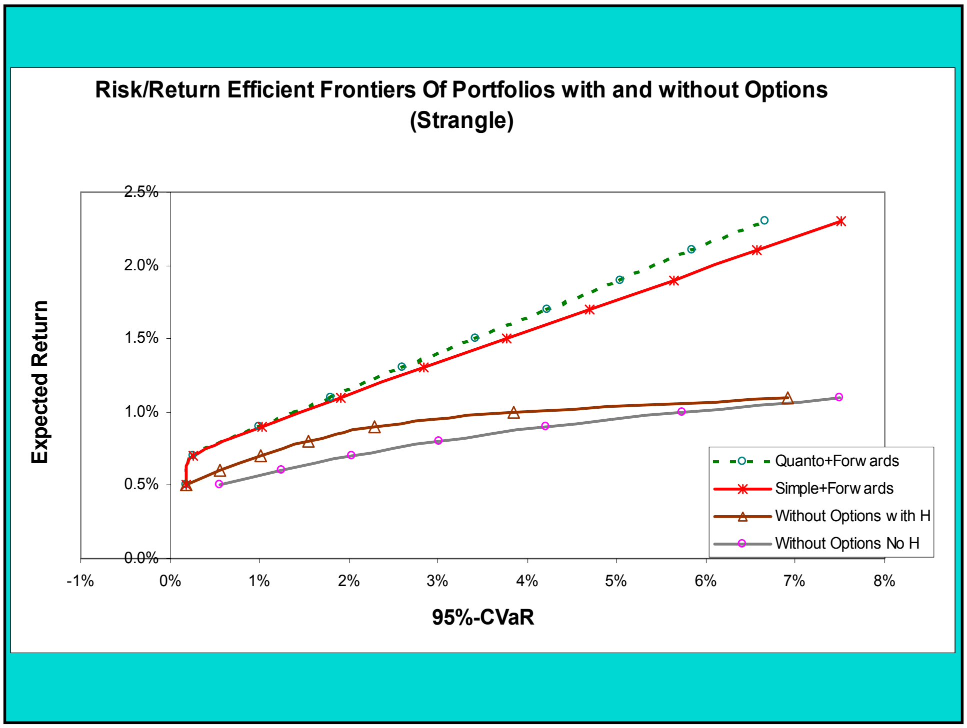
1 Long Call and 2 Long Puts
With the same exercise price X
and the same maturity

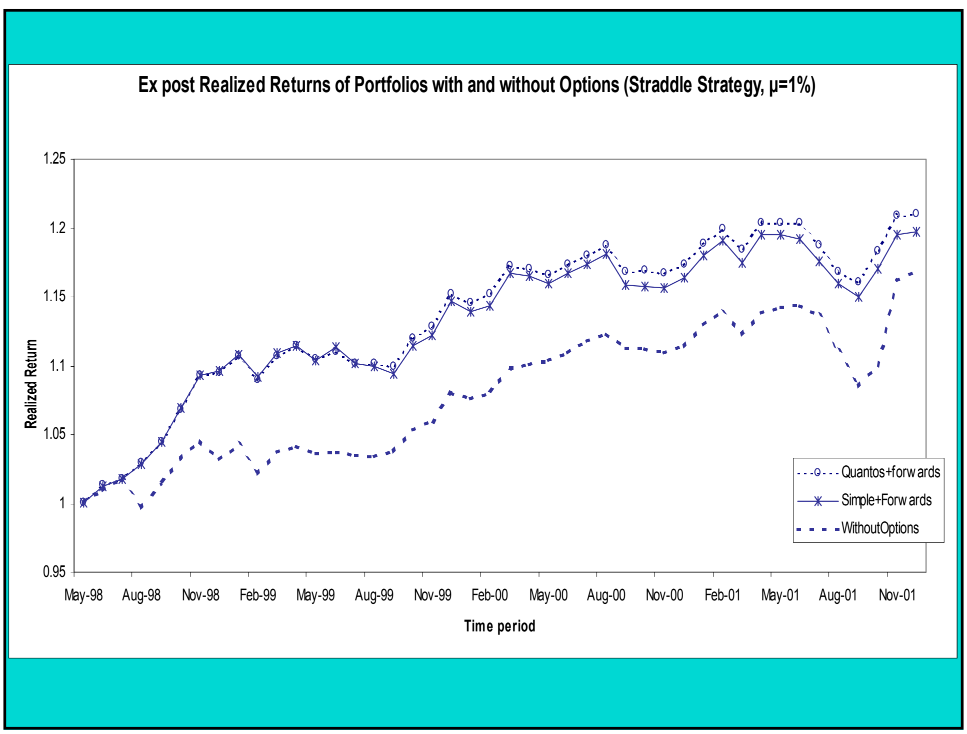
2 Long Call and 1 Long put
with the same exercise price X
and the same maturity

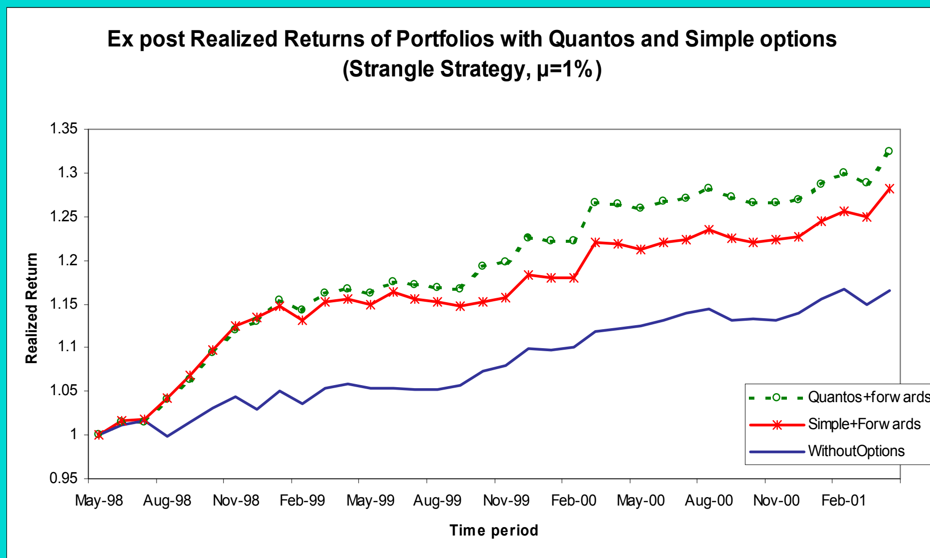



Risk/Return Efficient Frontiers Of Portfolios with various trading strategies of Quanto Options vs Portfolios Without Options
Expected Return
95%-CVaR
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
-1%
1%
3%
5%
7%
9%
11%
13%
15%
17%
Without Options
Straddle
Strangle
Strip
Strap


Ex post Realized Returns of Portfolios with different strategies of Options (μ=1%)
Realized Return
Time period
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
May-98
Aug-98
Nov-98
Feb-99
May-99
Aug-99
Nov-99
Feb-00
May-00
Aug-00
Nov-00
Feb-01
May-01
Aug-01
Nov-01
Strangle
Straddle
Strip
Strap
NoOptions


Risk/Return Efficient Frontiers Of Portfolios with and without Options
(Strangle)
Expected Return
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
-1%
0%
1%
2%
3%
4%
5%
6%
7%
8%
95%-CVaR
Quanto+Forwards
Simple+Forwards
Without Options with H
Without Options No H


Ex post Realized Returns of Portfolios with and without Options (Straddle Strategy, μ=1%)
Realized Return
1.25
1.2
1.15
1.1
1.05
1
0.95
May-98
Aug-98
Nov-98
Feb-99
May-99
Aug-99
Nov-99
Feb-00
May-00
Aug-00
Nov-00
Feb-01
May-01
Aug-01
Nov-01
Time period
Quantos+forwards
Simple+Forwards
WithoutOptions



Concluding Remarks:

- SPs potentially effective tools for international portfolio management
- Scenario generation methods provide effective means for representing uncertainty
- CVaR models constitute effective risk management tool
- Internalizing currency hedging decisions (via forward contracts) in the models improves ex-ante and ex-post results Positive value of integrative framework
- Introduction of derivatives leads to further performance improvements

  Particularly the integrated handling of market and currency risks (use of quantos)

HERMES Center of Excellence on Computational Finance & Economics

http://www.hermes.ucy.ac.cy

Working Paper Series:

(WP 01-23) *Journal of Banking and Finance*, 26(7), 2002