STANDARD
&POOR'S

Setting the Standard

Default Correlation: Empirical Evidence

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Agenda

- Do correlations matter ?
- Estimating default correlations empirically.
- Are equity correlations good proxies for asset correlations ?
- Correlations and the business cycle.
- Looking at correlations over longer horizons.

Do correlations matter?

- A lot of research has recently been devoted to default risk. Most of if has focused on the refinement of the estimation of default probabilities of **individual** firms.

- **But:** defaults do not occur independently. Macro-economic factors and industry specific events are common factors which impact on many firms and may lead to simultaneous defaults.

- **Example**: the current wave of defaults in the Telecom and Airline industries.

- At the portfolio level, dependencies between defaults are crucial and little is known about them.

Calculating empirical correlations.

- Consider the joint migration of two obligors from class i (say a BB rating) to class k (for example default).

- From a given group with N_i elements, one can create $N_i\ (N_i\text{-}1)/2$ different pairs. Denoting by $T_{i,k}$ the number of bonds migrating from this group to a given category k, one can obtain the joint probability using:

$$p_{i,i}^{k,k} = \sum_{t=1}^{n} \frac{N_i^t}{\sum_{s=1}^{n} N_i^s} \frac{T_{i,k}^t \left(T_{i,k}^t - 1\right)}{N_i^t \left(N_i^t - 1\right)}$$

- This is the estimator used by Lucas (1995) or Bahar and Nagpal (2001). Similar formulae can be derived for transitions to and from different classes.

Calculating empirical correlations.

- Although intuitive, this estimator has the drawback that it can generate spurious negative correlation when defaults are rare.

- We therefore propose to use:

$$p_{i,i}^{k,k} = \sum_{t=1}^{n} \frac{N_i^t}{\sum_{s=1}^{n} N_i^s} \frac{\left(T_{i,k}^t\right)^2}{\left(N_i^t\right)^2},$$

as an estimator of joint probability. It corresponds to drawing pairs with replacement.

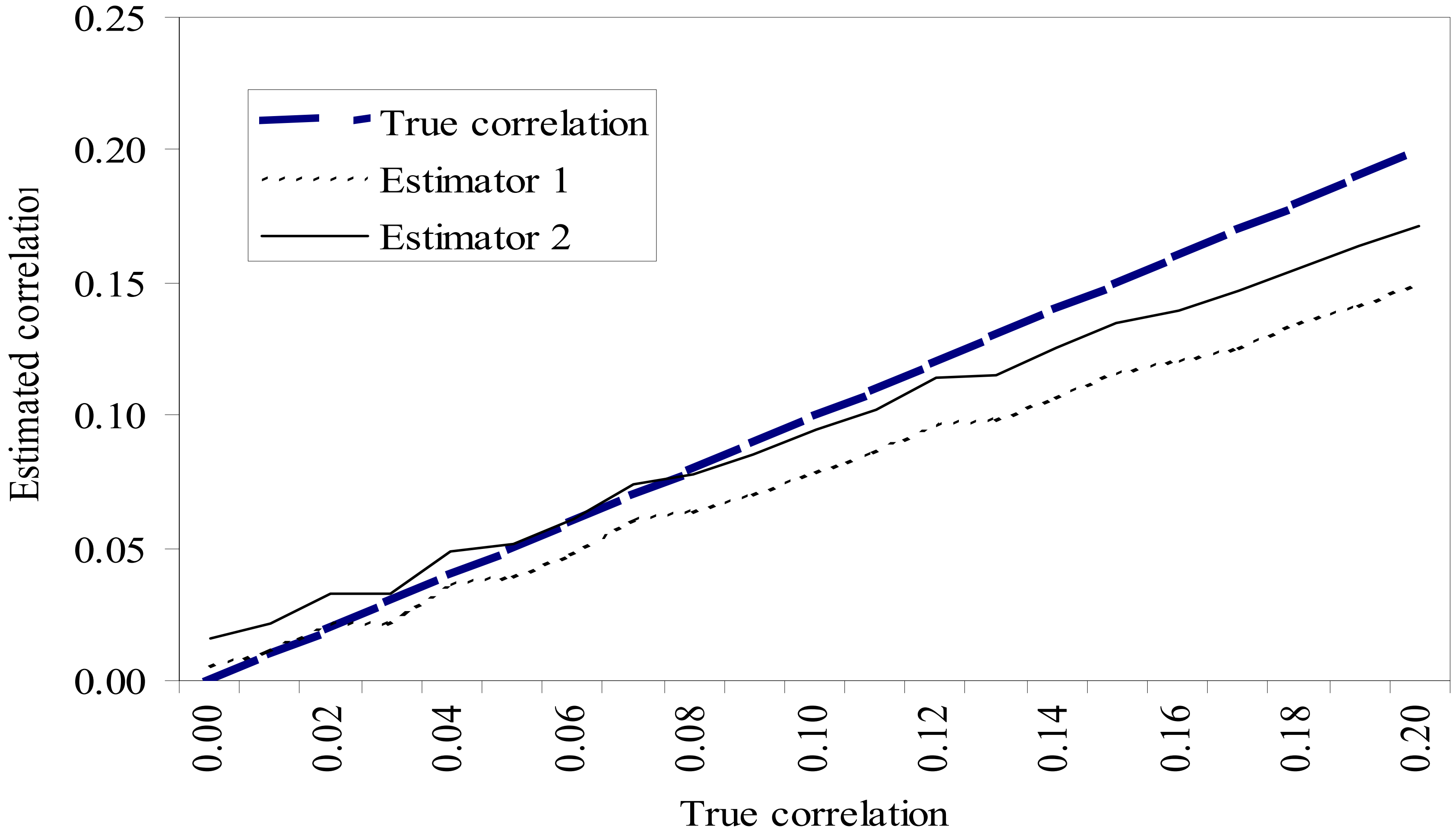
Calculating empirical correlations.

- Once we have estimated the joint probabilities, default correlations are calculated using the standard formula:

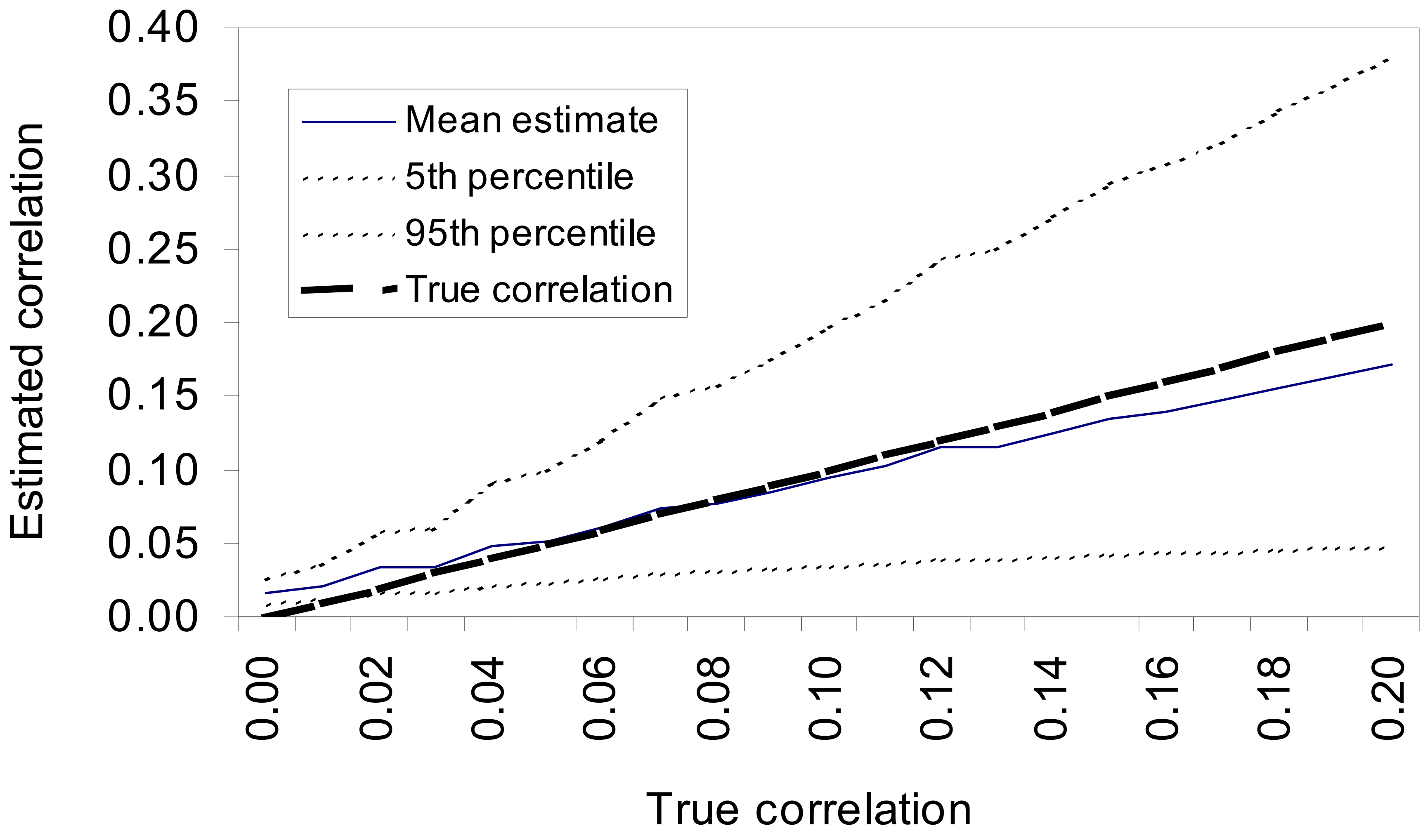
$$\rho_{i,j}^{k,l} = \frac{p_{i,j}^{k,l} - p_i^k p_j^l}{\sqrt{p_i^k (1 - p_i^k) p_j^l (1 - p_j^l)}}.$$

- Clearly, the correlation will be positive if the joint probability is larger than the product of the univariate probabilities.

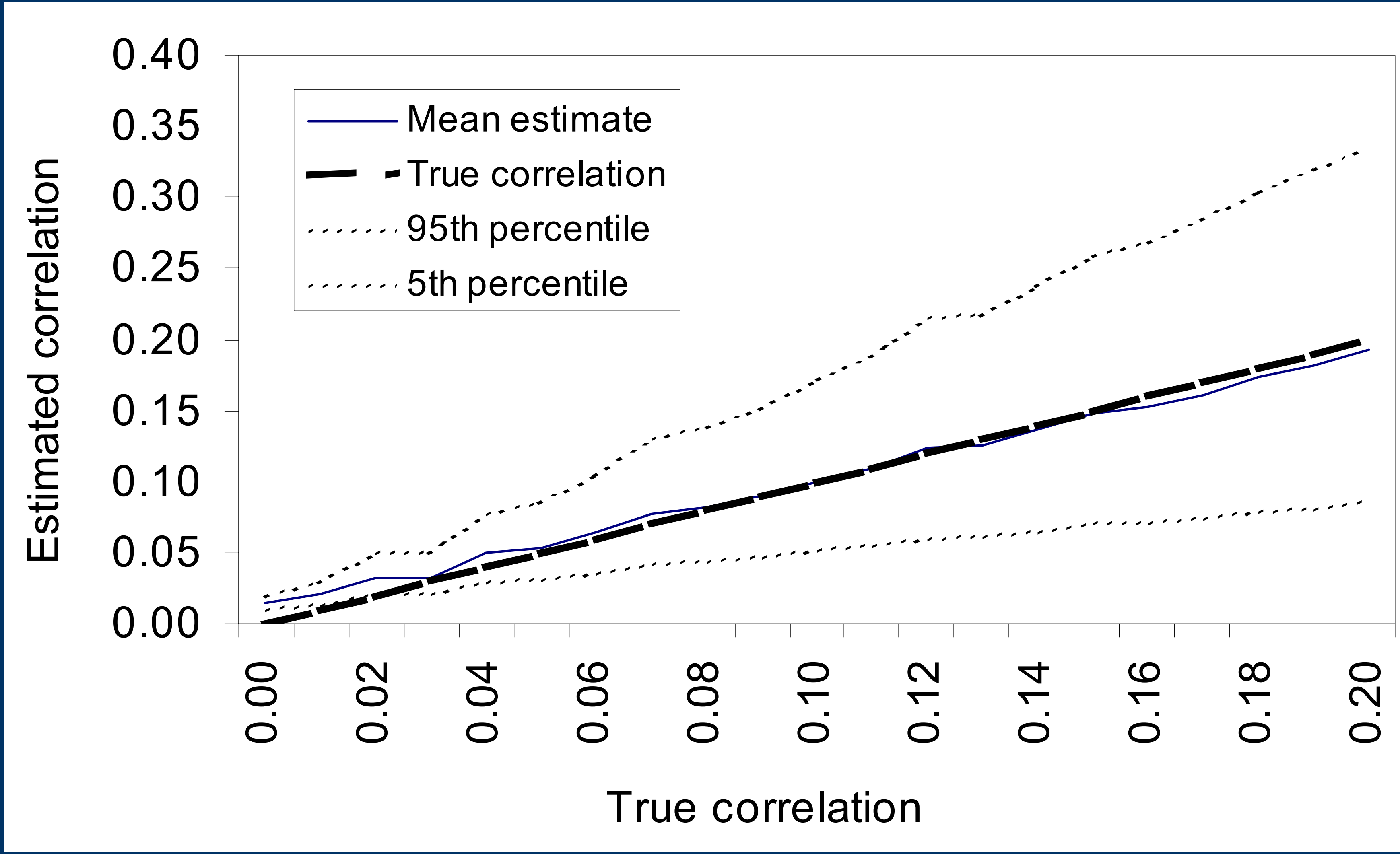
Performance of the estimators: 21 years of data.



Performance of the estimators: 21 years of data.



Performance of the estimators: 50 years of data.



The CreditPro database.

- Use Standard and Poor’s CreditPro 5.20 database.
- Features the last 21 years of default and transition experience for 9,769 companies rated by S&P since 1981.
- In this study we focus on the United States sub-sample. This comprises 6,907 firms and a total of 43,642 yearly observations.
- 764 defaults were recorded over the period 1981-2001.
- Ratings and in particular default data is very scarce outside the US.

Empirical correlations: US data

One year US default correlations - Non investment grade bonds 1981-2001

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Tele	Trans	Util
Auto	3.8%	1.3%	1.2%	0.4%	1.1%	1.6%	2.8%	-0.5%	1.0%	3.9%	1.3%	0.5%
Cons	1.3%	2.8%	-1.4%	1.2%	2.8%	1.6%	1.8%	1.1%	1.3%	3.2%	2.7%	1.9%
Ener	1.2%	-1.4%	**6.4%**	-2.5%	-0.5%	0.4%	-0.1%	-1.6%	-1.0%	-1.4%	-0.1%	0.7%
Fin	0.4%	1.2%	-2.5%	**5.2%**	2.6%	0.1%	0.4%	3.0%	1.6%	3.7%	1.5%	4.5%
Build	1.1%	2.8%	-0.5%	2.6%	**6.1%**	1.2%	2.3%	1.8%	2.3%	**6.5%**	4.2%	1.3%
Chem	1.6%	1.6%	0.4%	0.1%	1.2%	3.2%	1.4%	-1.1%	1.1%	2.8%	1.1%	1.0%
HiTec	2.8%	1.8%	-0.1%	0.4%	2.3%	1.4%	3.3%	0.0%	1.4%	4.7%	1.9%	1.0%
Insur	-0.5%	1.1%	-1.6%	3.0%	1.8%	-1.1%	0.0%	**5.6%**	1.2%	-2.6%	2.3%	1.4%
Leis	1.0%	1.3%	-1.0%	1.6%	2.3%	1.1%	1.4%	1.2%	2.3%	4.0%	2.3%	0.6%
Tele	3.9%	3.2%	-1.4%	3.7%	**6.5%**	2.8%	4.7%	-2.6%	4.0%	**10.7%**	3.2%	-0.8%
Trans	1.3%	2.7%	-0.1%	1.5%	4.2%	1.1%	1.9%	2.3%	2.3%	3.2%	4.3%	-0.2%
Util	0.5%	1.9%	0.7%	4.5%	1.3%	1.0%	1.0%	1.4%	0.6%	-0.8%	-0.2%	**9.4%**

Correlations above 5% are in bold.

Factor model of credit risk

- One of the most popular classes of credit risk models is the so-called factor-based approach.

- The rating transition process is the outcome of the realisation of systematic (macro, industry shocks) and idiosyncratic factors.

- Assume e.g. that the driving factor to be the value of the firm's assets. When this value falls below some critical threshold, default is triggered.

- A_j = latent variable driving default and migration for firm j.

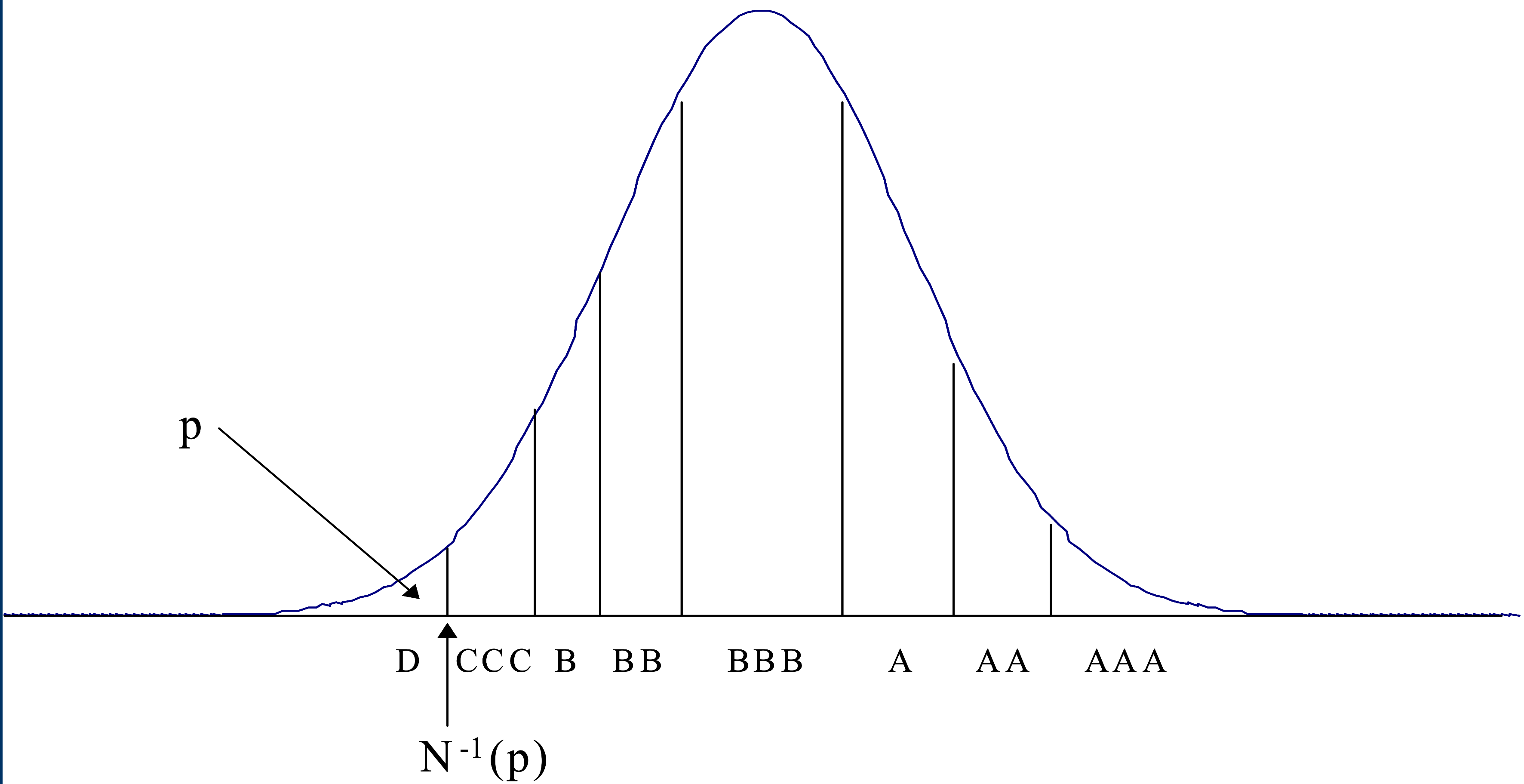
$$A_j = \rho_j C + \sqrt{1 - \rho_j^2}\,\varepsilon_j$$

Factor model of credit risk

- A set of thresholds is chosen such that when the value of the latent variable falls between two thresholds, the firm is assigned a given rating.
- The joint probability of two firms defaulting is therefore given by the probability that both their latent variables end up below the default thresholds.
- Given some standard assumptions, one can map the default correlation to the correlation between firms' asset values.

Factor model of credit risk

Calibrating transition probabilities using factor model



Are equity correlations good proxies for asset correlations?

- It has become market practice to use equity correlation as a proxy for asset correlation.
- Using a factor-model of credit risk, one can then derive default correlations.
- The question is: **do these default correlations resemble those calculated empirically**?
- To test this, we gathered a sample of over 1100 firms from S&P's 12 industry categories and calculated average equity correlations across and within industries.

Are equity correlations good proxies for asset correlations?

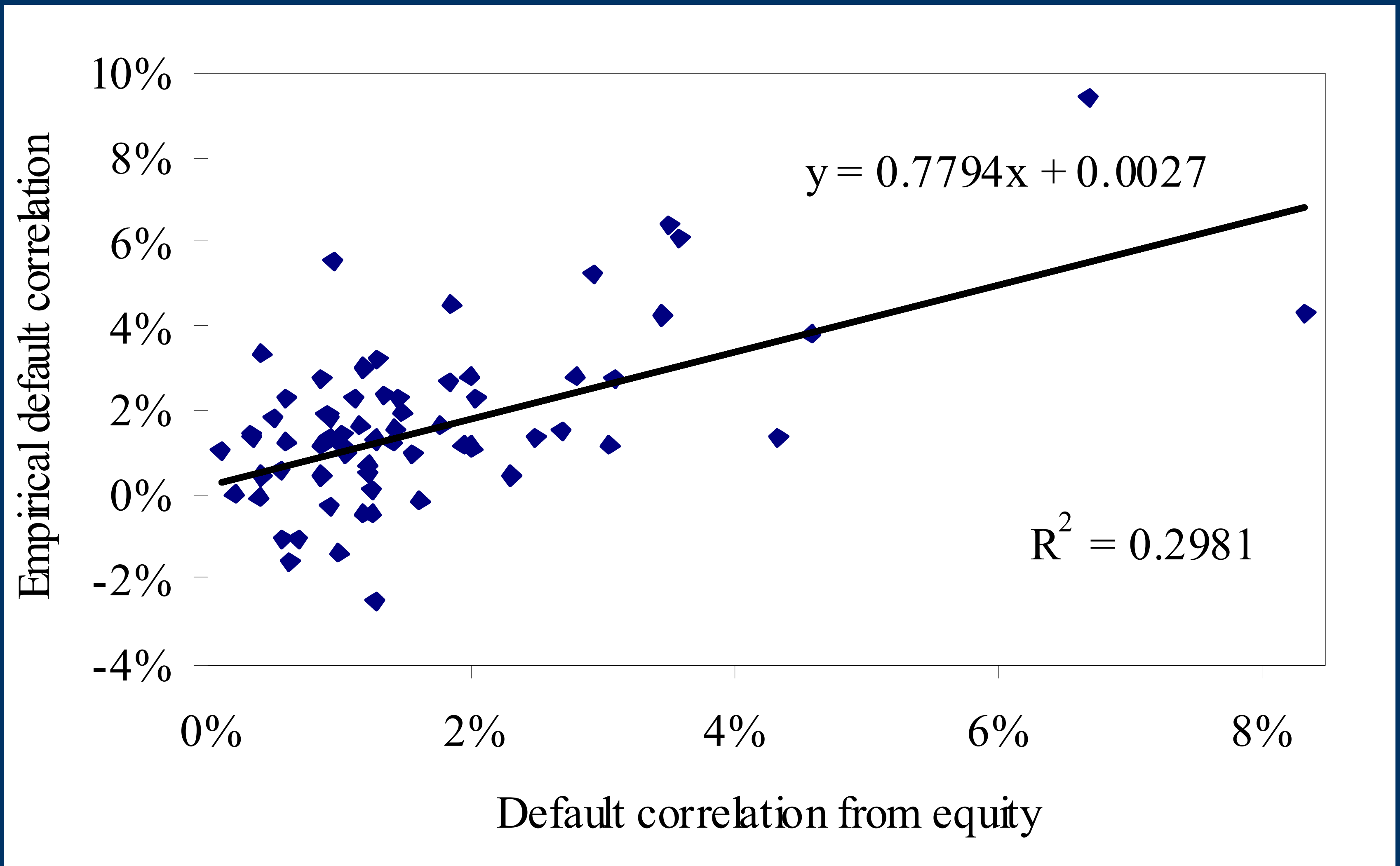
US default correlations - Non investment grade bonds 1981-2001

Calculated from equity correlations using a factor model.

One year horizon

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	4.6%	2.5%	1.9%	2.3%	3.1%	1.8%	0.8%	1.2%	1.6%	4.3%	1.2%
Cons	2.5%	2.8%	1.0%	2.0%	2.0%	1.4%	0.5%	1.0%	1.3%	3.1%	1.5%
Ener	1.9%	1.0%	3.5%	1.3%	1.2%	0.9%	0.4%	0.6%	0.7%	1.6%	1.2%
Fin	2.3%	2.0%	1.3%	2.9%	1.8%	1.2%	0.4%	1.2%	1.1%	2.7%	1.8%
Build	3.1%	2.0%	1.2%	1.8%	3.6%	1.4%	0.6%	0.9%	1.1%	3.5%	0.9%
Chem	1.8%	1.4%	0.9%	1.2%	1.4%	1.3%	0.3%	0.6%	0.8%	2.0%	1.0%
HiTec	0.8%	0.5%	0.4%	0.4%	0.6%	0.3%	0.4%	0.2%	0.3%	0.9%	0.1%
Insur	1.2%	1.0%	0.6%	1.2%	0.9%	0.6%	0.2%	0.9%	0.6%	1.4%	1.0%
Leis	1.6%	1.3%	0.7%	1.1%	1.1%	0.8%	0.3%	0.6%	1.3%	2.0%	0.5%
Trans	4.3%	3.1%	1.6%	2.7%	3.5%	2.0%	0.9%	1.4%	2.0%	8.3%	0.9%
Util	1.2%	1.5%	1.2%	1.8%	0.9%	1.0%	0.1%	1.0%	0.5%	0.9%	6.7%

Are equity correlations good proxies for asset correlations?



Are equity correlations good proxies for asset correlations?

- Equity correlations provide, at best, a **very noisy indicator** of default correlations.
- Disappointing result but maybe not surprising: equity returns incorporate a lot of noise (bubbles etc.) which are not related to the firms' fundamentals.
- Equity-based default correlations are very rarely (never in our sample) negative while empirical default correlations can be.
- Default correlations derived from equities have a similar order of magnitude as empirical correlations. (they are slightly higher)

Correlation and the business cycle.

- Macro-economic factors are the main drivers of credit losses at the portfolio level.
- The increase in default rates during recessions is well documented.
- **How do correlations change in expansions/recessions ?**
- **How do these changes impact on portfolio losses (CreditVaR)?**

Decomposing the Credit VaR

- Calculate the value at risk due to default (Credit VaR) on a fictitious corporate bond portfolio with:

  - identical position in all bonds ($1),

  - same default probability for all bonds,

  - same pairwise default correlation for all bonds.

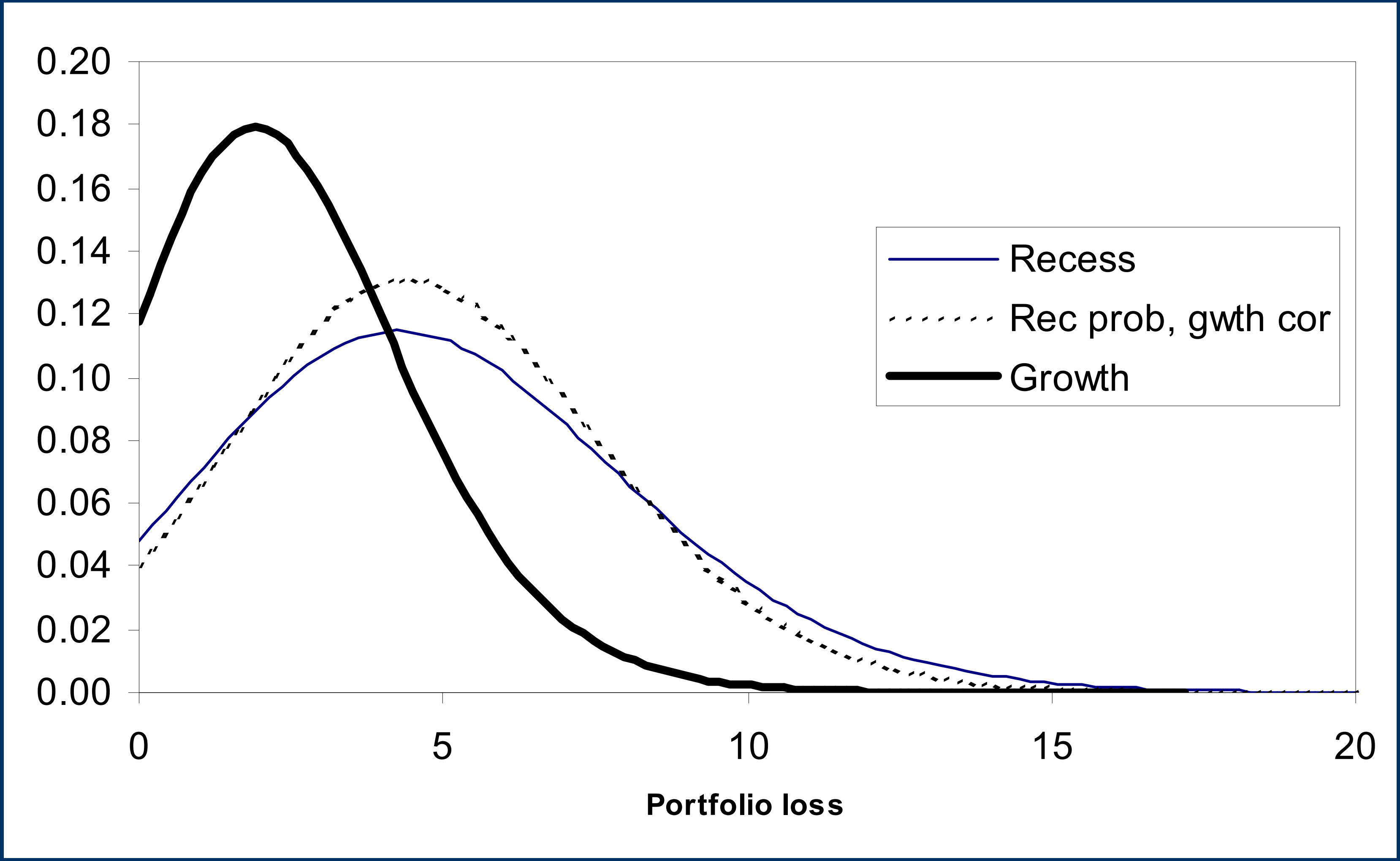
- Consider 3 scenarios:

  1) *growth*: default probability and correlation = average values in expansion.

  2) *recession*: default probability and correlation = average values in recession.

  3) *hybrid*: default probability = recession value, correlation = expansion.

Correlation and the business cycle.



Relative impact of correlation.

- Calculate the Credit VaR at various standard levels of confidence: 95%, 99%, 99.7% and 99.9% for our three scenarios.

- The further in the tail we look, the larger the relative impact of correlations.

CreditVaR in growth and recession

	95% VaR	99% VaR	99.7% VaR	99.9% VaR
Growth	6.8	9.0	10.4	11.8
Rec prob, gwth corr	9.6	11.8	13.4	14.2
Recession	10.8	13.6	15.4	17.2
Correlation contribution	30%	39%	40%	56%

- Correlation becomes the main driver of Credit VaR in the tails.

Correlation over longer horizons.

- So far, we have only considered the one-year horizon.
- This corresponds to the usual horizon for calculating VaR but not to the typical investment horizon of banks and asset managers.
- **What happens to correlations when we extend the horizon to 3 or 5 years ?**
- **Can a factor model of credit risk with constant correlation match the “term structure of correlation” empirically observed ?**

One-year empirical default correlation.

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	3.8%	1.3%	1.2%	0.4%	1.1%	1.6%	2.8%	-0.5%	1.0%	1.3%	0.5%
Cons	1.3%	2.8%	-1.4%	1.2%	2.8%	1.6%	1.8%	1.1%	1.3%	2.7%	1.9%
Ener	1.2%	-1.4%	6.4%	-2.5%	-0.5%	0.4%	-0.1%	-1.6%	-1.0%	-0.1%	0.7%
Fin	0.4%	1.2%	-2.5%	5.2%	2.6%	0.1%	0.4%	3.0%	1.6%	1.5%	4.5%
Build	1.1%	2.8%	-0.5%	2.6%	6.1%	1.2%	2.3%	1.8%	2.3%	4.2%	1.3%
Chem	1.6%	1.6%	0.4%	0.1%	1.2%	3.2%	1.4%	-1.1%	1.1%	1.1%	1.0%
HiTec	2.8%	1.8%	-0.1%	0.4%	2.3%	1.4%	3.3%	0.0%	1.4%	1.9%	1.0%
Insur	-0.5%	1.1%	-1.6%	3.0%	1.8%	-1.1%	0.0%	5.6%	1.2%	2.3%	1.4%
Leis	1.0%	1.3%	-1.0%	1.6%	2.3%	1.1%	1.4%	1.2%	2.3%	2.3%	0.6%
Trans	1.3%	2.7%	-0.1%	1.5%	4.2%	1.1%	1.9%	2.3%	2.3%	4.3%	-0.2%
Util	0.5%	1.9%	0.7%	4.5%	1.3%	1.0%	1.0%	1.4%	0.6%	-0.2%	9.4%

Three-year empirical default correlation.

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	6.1%	0.9%	5.1%	-1.4%	2.8%	6.4%	3.6%	-0.1%	2.3%	2.1%	3.0%
Cons	0.9%	3.7%	-4.1%	0.4%	3.5%	2.1%	2.4%	2.6%	3.1%	4.1%	3.1%
Ener	5.1%	-4.1%	13.0%	-7.0%	-1.5%	4.9%	0.9%	-3.5%	-3.2%	-2.3%	2.0%
Fin	-1.4%	0.4%	-7.0%	12.9%	8.3%	-1.2%	1.1%	7.9%	5.3%	5.5%	11.1%
Build	2.8%	3.5%	-1.5%	8.3%	10.7%	3.3%	4.1%	6.6%	6.7%	7.7%	4.6%
Chem	6.4%	2.1%	4.9%	-1.2%	3.3%	9.5%	4.8%	-1.1%	4.7%	2.4%	0.7%
HiTec	3.6%	2.4%	0.9%	1.1%	4.1%	4.8%	4.9%	1.0%	3.2%	3.8%	2.9%
Insur	-0.1%	2.6%	-3.5%	7.9%	6.6%	-1.1%	1.0%	6.5%	4.5%	5.1%	3.2%
Leis	2.3%	3.1%	-3.2%	5.3%	6.7%	4.7%	3.2%	4.5%	6.7%	6.4%	3.3%
Trans	2.1%	4.1%	-2.3%	5.5%	7.7%	2.4%	3.8%	5.1%	6.4%	7.2%	2.9%
Util	3.0%	3.1%	2.0%	11.1%	4.6%	0.7%	2.9%	3.2%	3.3%	2.9%	12.7%

Five-year empirical default correlation.

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	10.6%	2.1%	8.5%	-0.3%	3.1%	9.9%	5.7%	2.7%	3.4%	8.3%	3.7%
Cons	2.1%	7.1%	-7.8%	1.3%	5.3%	4.7%	3.2%	4.2%	7.0%	9.4%	5.0%
Ener	8.5%	-7.8%	21.8%	-9.5%	-6.3%	5.0%	4.5%	-1.2%	-7.2%	1.5%	5.2%
Fin	-0.3%	1.3%	-9.5%	19.3%	15.1%	1.8%	4.2%	9.1%	10.0%	14.8%	12.5%
Build	3.1%	5.3%	-6.3%	15.1%	14.3%	5.2%	4.5%	7.6%	11.7%	13.3%	8.0%
Chem	9.9%	4.7%	5.0%	1.8%	5.2%	14.6%	3.4%	1.9%	7.2%	6.5%	0.7%
HiTec	5.7%	3.2%	4.5%	4.2%	4.5%	3.4%	5.5%	3.8%	3.4%	6.0%	5.6%
Insur	2.7%	4.2%	-1.2%	9.1%	7.6%	1.9%	3.8%	5.8%	6.9%	7.3%	5.1%
Leis	3.4%	7.0%	-7.2%	10.0%	11.7%	7.2%	3.4%	6.9%	12.6%	15.1%	6.1%
Trans	8.3%	9.4%	1.5%	14.8%	13.3%	6.5%	6.0%	7.3%	15.1%	13.8%	6.9%
Util	3.7%	5.0%	5.2%	12.5%	8.0%	0.7%	5.6%	5.1%	6.1%	6.9%	12.1%

Correlation over longer horizons.

- Default correlations increase in the horizon.
- A constant *asset* correlation cannot replicate the extent of this increase.
- Using equity correlation without adjusting for the horizon is clearly insufficient.
- Need to take into account the term structure of correlations.

Conclusion.

- Default correlations increase in the horizon.
- A constant *asset* correlation cannot replicate the extent of this increase.
- Using equity correlation without adjusting for the horizon is clearly insufficient.
- We advocate the use of empirical default correlation to benchmark internal models.